

**Exercises 8:** 3, 4, 7, 18, 40, 46

**Exercises 9:** 3, 5, 9, 12

**Additional exercises:**

1. Let  $G = \mathbb{R}^3$  be the group of real 3-tuples with component-wise addition (vector addition).

(a) What geometric object is the set

$$H = \{(x, y, z \in \mathbb{R}^3 \mid 3(x - 1) + 2(y + 5) - (z + 1) = 6\}?$$

Prove that  $H$  is a subgroup of  $G$ .

- (b) The cyclic subgroup generated by  $(0, 0, 0)$  is the trivial subgroup of  $G$ , but all other cyclic subgroups are more interesting. Let  $K$  be the cyclic subgroup generated by a nonidentity element  $(a, b, c)$ . Give an algebraic and geometric description of  $K$ .
- (c) Let  $L$  be the line through the origin and the point  $(2, 3, 5)$ . Is  $L$  a subgroup of  $G$ ? If so, is it a cyclic subgroup of  $G$ ? (It is easiest to check if you come up with an algebraic description of  $L$ .) Be sure to justify your answers.