Exercises 10: 4, 6, 12, 16, 20–24, 28, 29, 39, 40, 43

Exercises 11: 16, 18, 23, 39, 47, 49, 53, 54

Additional exercises:

1. Prove that every group is isomorphic to a subgroup of a permutation group using the following steps. (This is called Cayley's Theorem.) Let G be a group, and for each $g \in G$, let $\lambda_g \colon G \to G$ be the map defined by $\lambda_g(h) = gh$.
   (a) Prove that $\lambda_g \in S_G$. (Recall that S_G is the group of permutations of the set G.)
   (b) Let $\phi \colon G \to S_G$ be defined by $\phi(g) = \lambda_g$. Prove that ϕ is an isomorphism between G and its image, $\phi(G)$.
   (c) Conclude that G is isomorphic to a subgroup of S_G.
2. Let G_1 and G_2 be groups.
   (a) Prove that the map $\pi \colon G_1 \times G_2 \to G_1$ defined by $\pi((g, h)) = g$ is a surjective homomorphism but not an isomorphism.
   (b) Prove that the map $\iota \colon G_1 \to G_1 \times G_2$ defined by $\iota(g) = (g, e_2)$ (where e_2 is the identity element of G_2) is an injective homomorphism but not an isomorphism.
3. Let $G_1, \ldots, G_n$ be groups, and let $G = \prod_{i=1}^{n} G_i$. Prove that for each i, G_i is isomorphic to a subgroup of G.
4. Prove that every group of prime order is cyclic.

Challenge problem (optional, not to be turned in):

1. Let G be a group with order p^n for some prime p and some $n \in \mathbb{Z}^+$. Prove that G contains an element of order p.