

Note: Most of these problems are from Homework 7a, so you may have done them already!

**Exercises 14:** 27, 38, 39

**Exercises 15:** 35, 36, 39, 42

**Additional exercises:**

1. Prove that the quotient group of a cyclic group is cyclic.
2. Let  $G = D_4$ . Show that  $\langle \tau, \rho^2 \rangle$  is a normal subgroup of  $D_4$  and  $\langle \tau \rangle$  is a normal subgroup of  $\langle \tau, \rho^2 \rangle$ , but that  $\langle \tau \rangle$  is *not* a normal subgroup of  $D_4$ . (Note: This proves that the relation “is a normal subgroup of” is not transitive.)
3. Let  $\phi: G \rightarrow G'$  be a homomorphism. Prove that if  $N$  is a normal subgroup of  $G$ , then  $\phi(N)$  is normal subgroup of  $\phi(G)$ . [Notice that I stated this incorrectly in class, even though I was asked a question about it!  $\phi(N)$  is not necessarily a normal subgroup of  $G'$ . Try to think of an example where this is the case (you do not have to turn this in).] Prove also that if  $N'$  is a normal subgroup of  $\phi(G)$ , then  $\phi^{-1}(N')$  is a normal subgroup of  $G$ .