1. Write the element $\sigma = (1\,9\,3)(2\,5\,8\,4\,7\,6) \in S_9$ as the product of transpositions. Is $\sigma \in A_9$?

2. Find the order of $(1\,3\,5\,4\,2)(1\,2\,4\,5\,3)(1\,4)(2\,5)$ in S_5.

3. Find all possible non-trivial homomorphisms $\phi\colon S_3 \to \mathbb{Z}_4$.

4. Let $\phi\colon G \to G'$ be a homomorphism where $|G| = 9$. Find $|\ker(\phi)|$ if ϕ is:
   (a) trivial
   (b) injective
   (c) neither

5. Consider the dihedral group D_6, and recall that in D_6, $\rho\tau = \tau\rho^{-1}$. Show that:
   (a) $\langle \rho^3 \rangle \lhd D_6$, and
   (b) $D_6/\langle \rho^3 \rangle \simeq S_3$.

6. Is $\langle (1\,2\,3) \rangle$ a normal subgroup of S_4?

7. Let G be an abelian group. Show that the map $\phi\colon G \to G$ defined by $\phi(x) = x^{-1}$ for all $x \in G$ is an automorphism

8. Let G be a group, $H \lhd G$, and ϕ an automorphism of G. Show that $\phi(H)$ is a normal subgroup of G.

9. Let $Z(G)$ be the center of a group G. Show that:
   (a) $Z(G) \lhd G$, and
   (b) if $G/Z(G)$ is cyclic, then G is abelian.

10. Let $H, K \leq G$. Then $HK = \{hk \mid h \in H, k \in K\}$. Show that HK is a subgroup of G if and only if $HK = KH$.

11. Let $K \lhd G$ and let H be a subgroup of G. Show that $K \cap H \lhd H$.

12. Let G be a group with $|G| = p^2$, where p is prime. Show that every proper subgroup of G is cyclic.

13. Show that the non-zero elements of $\mathbb{Z}_p$, where p is prime, form a group under multiplication.