Give a non-trivial example of each of the following objects. All are possible. If you can come up with more than one example of some, that would be good practice.

1. A finite ring of matrices.
2. A subring of $\mathbb{C}$ which does not have unity.
3. A matrix ring which does not have unity.
4. Three units in $M_3(\mathbb{Z})$.
5. A ring homomorphism $\phi : \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$.
6. A finite ring with at least 3 zero divisors.
7. A ring whose only units are 1 and -1.
8. A solution of the equation $x^2 + 5x + 6$ in Z_{12} other than -2 or -3 (which come from factoring).
9. A finite field with at least 20 elements.
10. A zero divisor of the ring $\mathbb{Z}_5 \times \mathbb{Z}_7$.
11. A polynomial ring which is an integral domain.
12. A ring without unity that has no zero divisors.
13. A polynomial ring which is not an integral domain.
14. An integral domain whose field of quotients is $\mathbb{R}$.
15. An ideal of $\mathbb{Z}_3 \times \mathbb{Z}_4$ which is not a prime ideal.
16. A principal ideal of $\mathbb{Z}_3 \times \mathbb{Z}_4$ which is a prime ideal.
17. A maximal ideal of $\mathbb{R}[x]$.
18. A ring which has no proper nontrivial maximal ideals.
19. A ring R which is an integral domain but not a field, and an ideal I of R such that R/I is not a field.
20. A ring R which is an integral domain but not a field, and an ideal I of R such that R/I is a field.

21. A polynomial ring R which is an integral domain, and an ideal I of R such that R/I has zero divisors.

22. A ring R which is not an integral domain, and an ideal I of R such that R/I is an integral domain.

23. Two non-isomorphic rings which each contain 3 elements.

24. A non-trivial ring homomorphism $\phi\colon \mathbb{Z}[x] \to \mathbb{Z}_3[x]$.

25. A nontrivial ring homomorphism $\phi\colon \mathbb{Z}[x] \to \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$.

26. A polynomial in $\mathbb{Z}[x]$ which has 4 terms and is irreducible over $\mathbb{Q}$ by Eisenstein's Criteria.

27. A polynomial in $\mathbb{Z}[x]$ which has 4 terms and is irreducible over $\mathbb{Q}$, but Eisenstein's Criteria do not apply.

28. An irreducible quadratic polynomial in $\mathbb{Z}_5[x]$.

29. Two different proper subgroups A and B of D_4 such that $A \triangleleft B$ and $B \triangleleft D_4$, but A is not a normal subgroup of D_4.

30. Two subgroups A and B of $G = \mathbb{Z}_2 \times \mathbb{Z}_4$ such that $G/A \simeq \mathbb{Z}_4$ and $G/B \simeq \mathbb{Z}_2 \times \mathbb{Z}_2$.

31. A non-trivial homomorphism $\phi\colon \mathbb{Z}_{12} \times D_4 \to \mathbb{Z}_4$.

32. A subgroup of $S_3 \times \mathbb{Z}_4$ which has exactly 8 elements.

33. An infinite group G and a subgroup H such that there are infinitely many left cosets of H in G.

34. An element of $\mathbb{Z}_4 \times \mathbb{Z}_5 \times \mathbb{Z}_6$ which has order 10 and does not have a 0 in any component.

35. A non-abelian group with at least 6 elements of order 5.

36. A pair of zero divisors in the ring $\mathbb{Z}_5 \times M_2(\mathbb{Z})$.

37. An extension of $\mathbb{Q}$ which is algebraic of degree 4.

38. Given an example of a commutative ring without zero-divisors that is not an integral domain.

39. Find a ring R and two elements $a, b \in R$ such that a and b are zero-divisors and $a + b$ is a unit.