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Short conjugators in higher rank lamplighter groups

I am a third year Ph.D. student working under Tullia Dymarz. I am currently working on bounding the length of short conjugators in higher rank lamplighter groups, using the geometric properties of their Cayley graphs.

A lamplighter group can be defined as a wreath product, $\Gamma_2(q) = \mathbb{Z}_q \wr \mathbb{Z}$, and its Cayley graph, called a Diestel-Leader graph, is the horocyclic product of two $(q+1)$-valent trees, T_1 and T_2; that is, $DL_2(q) = \{(x,y) | x \in T_1,\, y \in T_2,\, h(x) + h(y) = 0\} \subset T_1 \times T_2$, where h is a height function $h : T_i \to \mathbb{Z}$. Lamplighter groups have solvable conjugacy problem, and Sale gives a geometric method for finding a linear bound on the length of short conjugators in [1]. That is, given any two conjugate elements $u, v \in \Gamma_2(q)$, with $\|u\| + \|v\| = n$, one can find a conjugator whose length is bounded above by a linear function of n. His method relies on the geometry of the Diestel-Leader graph and the work of Stein and Taback, [2], who give a geometric method for bounding the lengths of elements in terms of the geometry of the Diestel-Leader graphs.

I am interested in extending Sale's methods to higher rank lamplighter groups, which can be defined as the semidirect product $\Gamma_d(q) = \mathbb{Z}_q[x, (l_1 + x)^{-1}, \ldots, (l_{d-1} + x)^{-1}] \rtimes_\phi \mathbb{Z}^n$. The l_i are elements of a commutative ring of order q, with pairwise differences $l_i - l_j$ invertible, and $(a_1, \ldots, a_d) \in \mathbb{Z}^n$ acts by multiplication by $x^{a_1}(l_1 + x)^{a_2} \ldots (l_{d-1} + x)^{a_d}$. The Cayley graph of $\Gamma_d(q)$ is a Diestel-Leader graph $DL_d(q)$, which is the horocyclic product of d $(q+1)$-valent trees. By generalizing Sale's method for the case $d = 2$, I hope to find a linear bound on the length of short conjugators in these higher lamplighter groups.

References

[1] Sale, Andrew. *The length of conjugators in solvable groups and lattices of semisimple Lie groups.* University of Oxford Ph.D. Thesis.

[2] M. Stein and J. Taback. *Metric properties of Diestel-Leader groups.* The Michigan Mathematical Journal 62 (2013), no. 2, 365–386.