

Applications of Differential Equations

For each problem, set up a differential equation modeling the situation. Be sure to define your variables, state the units being used, and determine the initial condition. You do not need to solve.

1. A 100 gallon tank is full of pure water. Let pure water run into the tank at the rate of 2 gals/min. and a brine solution that has 0.5 lbs/gal. of salt run in at the rate of 2 gals/min. The mixture flows out of the tank through an outlet tube at the rate of 4 gals/min. Assuming perfect mixing, what is the amount of salt in the tank after t minutes?
2. A tank has pure water flowing into it at 12 l/min. The contents of the tank are kept thoroughly mixed, and the contents flow out at 10 l/min. Initially, the tank contains 10 kg of salt in 100 l of water. How much salt will there be in the tank after 30 minutes?
3. A large tank is filled to capacity with 100 gallons of pure water. Brine containing 3 pounds of salt per gallon is pumped into the tank at a rate of 4 gal/min. The well-mixed solution is pumped out of the tank at the rate of 5 gal/min. Find the amount of salt after 30 minutes.
4. A tank with 200 gallons of brine solution contains 40 lbs of salt. A concentration of 2 lb/gal is pumped in at a rate of 4 gal/min. The concentration leaving the tank is pumped out at a rate of 4 gal/min. How much salt is in the tank after 1 hour?
5. A 1000L tank starts out with 200L of fluid containing 10g/L of dye. Pure water is poured in at 20 L/min and the tank is being drained at a rate of 15L/min. Write the equation for the amount of dye in the tank at any time t .
6. The rate at which a certain drug is eliminated from the bloodstream is proportional to the amount of the drug in the bloodstream. A patient now has 45 mg of the drug in his bloodstream. The drug is being administered to the patient intravenously at a constant rate of 5 milligrams per hour. Write a differential equation modeling the situation.
7. Ten thousand dollars is deposited in a bank account with a nominal annual interest rate of 5% compounded continuously. No further deposits are made. Write a differential equation reflecting the situation if money is withdrawn continuously at a rate of \$4000 per year.
8. In the absence of predators the population of mosquitoes in a certain area would increase at a rate proportional to its current population and its population would double every three weeks. There are 250,000 mosquitoes in the area initially when a flock of birds arrives that eats 80,000 mosquitoes per week. How many mosquitoes remain after two weeks?
9. (Challenge!) A 1000 gallon holding tank that catches runoff from some chemical process initially has 800 gallons of water with 2 ounces of pollution dissolved in it. Polluted water flows into the tank at a rate of 3 gal/hr and contains 5 ounces/gal of pollution in it. A well mixed solution leaves the tank at 3 gal/hr as well. When the amount of pollution in the holding tank reaches 500 ounces the inflow of polluted water is cut off and fresh water will enter the tank at a decreased rate of 2 gal/hr while the outflow is increased to 4 gal/hr. Determine the amount of pollution in the tank at any time t .