Solution to 3/16 Example

Problem: Find the point on the intersection of the planes $y+2z=12$ and $x+y=6$ that is closest to the origin.

Solution: We want to minimize $f(x,y,z)=x^2+y^2+z^2$ (distance to origin squared) subject to the constraints $g(x,y,z)=y+2z=12$ and $h(x,y,z)=x+y=6$

$$\begin{cases} \nabla f = \lambda \cdot \nabla g + \mu \cdot \nabla h \\ y+2z=12 \\ x+y=6 \end{cases} \Rightarrow \begin{cases} 2x = \lambda \cdot 0 + \mu \cdot 1 \longrightarrow \mu = 2x \\ 2y = \lambda \cdot 1 + \mu \cdot 1 \\ 2z = \lambda \cdot 2 + \mu \cdot 0 \longrightarrow \lambda = z \\ y+2z=12 \\ x+y=6 \end{cases}$$

So, $\begin{cases} y = z+2x & (1) \\ y+2z=12 & (2) \\ x+y=6 \rightarrow y=6-x \end{cases}$

From (1): $6-x=z+2x$

$z=6-3x$

From (2): $6-x+2(6-3x)=12$

$6-x+12-6x=12$

$-7x=-6$

$x=6/7$

$x=6/7$; so $z=6-3(6/7)$

$z=24/7$

and $y=6-6/7$

$y=36/7$

$$\boxed{(6/7,\ 36/7,\ 24/7) \text{ is the closest point to the origin.}}$$