1. Let $f(x, y) = 4x^2 - 4xy + 4y^2$, and suppose $x = \cos t$ and $y = \sin t$. Find $\dfrac{\mathrm{d}f}{\mathrm{d}t}$ using the chain rule.

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2. Find the gradients of the following functions.
   (a) $f(x, y) = x^2 + y^2$
   (b) $f(x, y) = xy \ln(xy)$
   (c) $f(x, y) = \sqrt{x^2 + y^2}$
   (d) $f(x, y) = x^2 \sin(y)$

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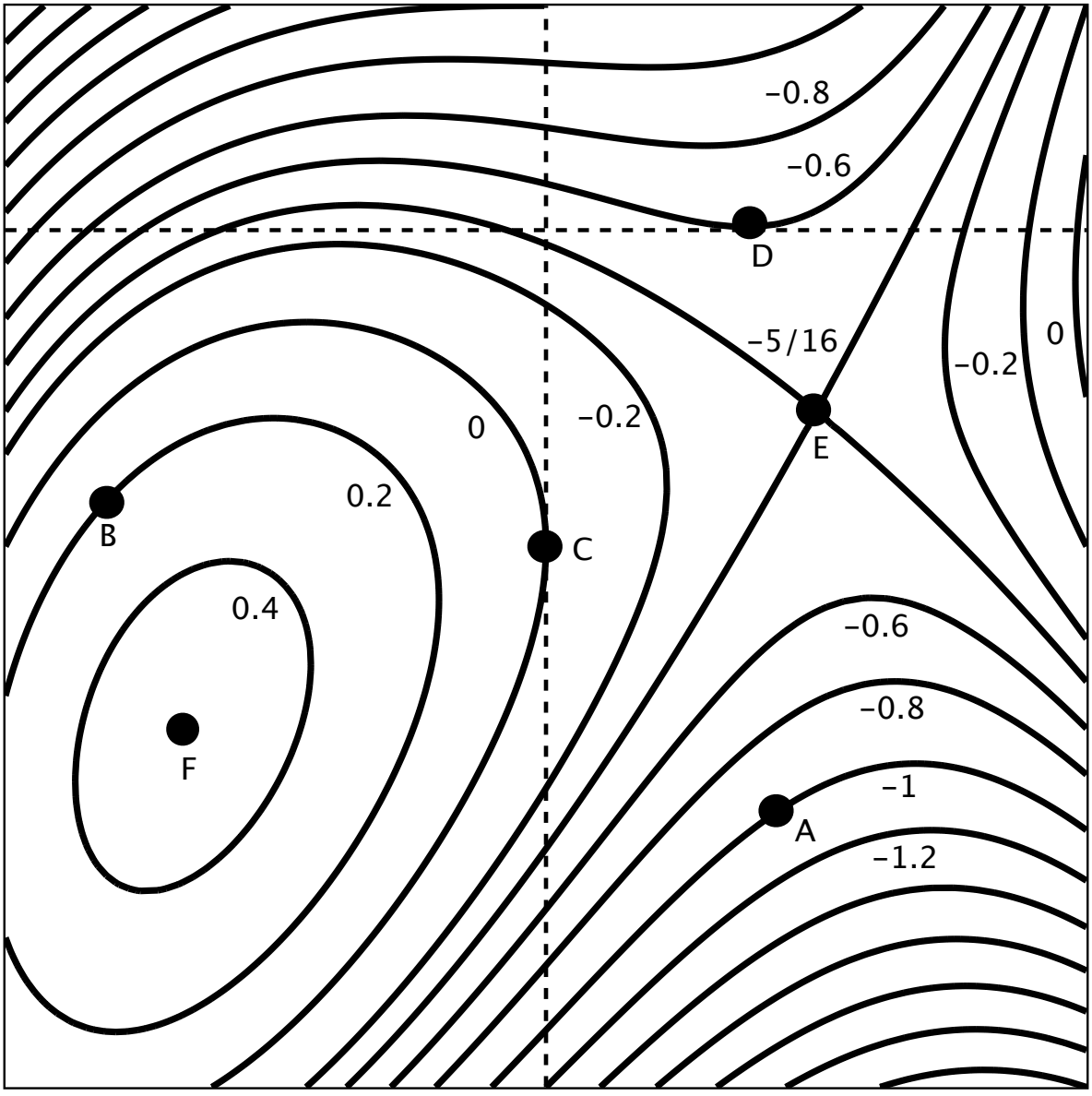
3. Let $f(x, y) = x^2 + y^2$. Find all the points on the level set $f(x, y) = 1$ where the gradient is parallel to $\vec{\mathbf{i}} + 2\vec{\mathbf{j}}$.

The following figure is a level set plot for a differentiable function $f(x, y)$. The horizontal axis (not shown) is the x-axis, and the vertical axis (not shown) is the y-axis. The dashed vertical line is the tangent line to the level set $f = 0$ at C. The dashed horizontal line is the tangent line to the level set $f = -6$ at D.



Using the figure, answer the questions on the following page.

1. Determine whether $f'_x(A)$ is positive, zero, or negative. Do the same for $f'_y(A)$.

2. Determine whether $f'_x(B)$ is positive, zero, or negative. Do the same for $f'_y(B)$.

3. The dashed vertical line is the tangent line to the level set $f = 0$ at C.
   (a) Determine whether $f'_x(C)$ is positive, zero, or negative.
   (b) Suppose you are standing at the point C. Describe what happens to f as you move directly North or South, starting from the point C. Does f increase, decrease, or stay the same?
   (c) Determine whether $f'_y(C)$ is positive, zero, or negative.

4. The dashed horizontal line is the tangent line to the level set $f = -6$ at D.
   (a) Suppose you are standing at the point D. Describe what happens to f as you move directly West or East, starting from the point D. Does f increase, decrease, or stay the same?
   (b) Determine whether $f'_x(D)$ is positive, zero, or negative. Do the same for $f'_y(D)$.

5. (a) Suppose you are standing at the point E. Describe what happens to f as you move directly North, South, West, or East. Does f increase, decrease, or stay the same?
   (b) Based on your observations in (a), what can you say about $f'_x(E)$ and $f'_y(E)$?

6. The level set passing through the point F is not really missing from the figure. What is it? Can you figure out what $f'_x(F)$ and $f'_y(F)$ are?

7. (a) Can you locate (just eyeball it) all the points P in the xy-plane for which $f'_y(P) = 0$? Do these points fit a simple pattern in the figure? What is interesting about this pattern?
   (b) Can you locate (just eyeball it) all the points Q in the xy-plane for which $f'_x(Q) = 0$? Do these points fit a simple pattern in the figure? How would you describe this pattern?

8. A mysterious sage from the Himalayas tells you that the range for the variable x in the figure is $0 \leq x \leq 2$, and the range for y is $0 \leq y \leq 2$. She also tells you that the function $f(x, y)$ whose level set plot is given in the figure is actually

$$f(x, y) = x^3 - 3x^2 + x + xy - y^2 + y.$$

(a) For the sage's function $f(x, y)$, compute f'_x and f'_y.

(b) Set $f'_y = 0$. Sketch the graph of your equation $f'_y = 0$ in the figure. Does this confirm your suspicion from 7(a)?

(c) Set $f'_x = 0$ and see if this confirms your suspicion from 7(b).

(d) Can we trust the sage now?

(e) Find the exact coordinates of the points E and F ... the sage will be impressed.
