1. Recall that we can transform between Cartesian (x and y) coordinates and polar (R and θ) coordinates with the following formulae

$$R(x,y) = \sqrt{x^2+y^2} \qquad \theta(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$$

$$x(R,\theta) = R\cos(\theta) \qquad y(R,\theta) = R\sin(\theta)$$

In this problem, if I'm telling you a point in polar coordinates, I'll write it as $(\cdot,\cdot)_{R,\theta}$.

(a) Plot the points $(1,0)_{R,\theta}$, $(1,\pi/2)_{R,\theta}$, $(1,\pi)_{R,\theta}$, and $(1,2\pi)_{R,\theta}$ in the xy-plane. To do this, use the formulas above to convert the R and θ values into x and y values.

(b) Express the point $(1,1)$ in polar coordinates.

(c) Express the equation $x^2+y^2=1$ in polar coordinates. Do the same with the equations $x=1$ and $y=-x+1$ and solve for R in both cases.

(d) Express the equation $R=\cos(\theta)$ in Cartesian coordinates. What is the graph of this equation?

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2. Suppose $f(R,\theta)$ is a function in polar coordinates. Using the chain rule and the above formulae, come up with a formula for computing $\frac{\partial f}{\partial x}$.

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3. Consider the equation $x^2+y^2=1$. Near the point $(1,0)$, can you express y as a function of x? How about x as a function of y? Do the same for the equation $x^2-y^2+y^3=0$ at the point $(0,1)$.

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4. Convert the equation $y=mx+b$ into polar coordinates. For which values of m and b is R a function of θ?

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5. Can you come up with something analogous to the "vertical line test" to detect if R is a function of θ?

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6. Let $f(x,y)=x^3+y^2\sin(x)$.

(a) Calculate the gradient of f.

(b) Find the equation of the plane tangent to the graph of f at the point $(x_0,y_0)=(1,0)$.

(c) What is the tangent vector of the plane you found?

(d) Using linear approximations of f, approximate $f(1.1,0.1)$ and $f(0.1,1.1)$. Should you use the same linear approximation formula for both of these points?

7. Is the quadratic form $f(x, y) = 3x^2 + 8xy - 3y^2$ definite, semi-definite, or indefinite? Draw its zero set and indicate where it's positive and negative.