1. Suppose that a differentiable function $f(x,y,z)$ satisfies

$$\vec{\nabla} f(8,4,12) = \begin{pmatrix} A \\ B \\ C \end{pmatrix}, \quad \vec{\nabla} f(12,27,3) = \begin{pmatrix} D \\ E \\ F \end{pmatrix}, \quad \vec{\nabla} f(12,32,2) = \begin{pmatrix} G \\ H \\ I \end{pmatrix}, \quad \vec{\nabla} f(12,27,6) = \begin{pmatrix} J \\ K \\ L \end{pmatrix}.$$

Furthermore suppose

$$x = u + v, \quad y = uv, \quad z = u/v.$$

Define $h(u,v) = f(x,y,z)$. Find $\dfrac{\partial h}{\partial u}(8,4)$ and $\dfrac{\partial h}{\partial v}(8,4)$. Also find $\dfrac{\partial h}{\partial u}(9,3)$ and $\dfrac{\partial h}{\partial v}(9,3)$.

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2. Let $f(x,y)$ be differentiable, and consider the polar variables r, θ.

(a) Find $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial \theta}$ in terms of r, θ, f_x, f_y.

(b) Use part (a) to simplify the following two expressions:

$$\cos\theta \frac{\partial f}{\partial r} - \frac{1}{r}\sin\theta \frac{\partial f}{\partial \theta} \qquad\qquad \sin\theta \frac{\partial f}{\partial r} + \frac{1}{r}\cos\theta \frac{\partial f}{\partial \theta}$$

(c) Use what you got in part (b) to express $\|\vec{\nabla} f\|$ in terms of $r, \theta, \dfrac{\partial f}{\partial r}, \dfrac{\partial f}{\partial \theta}$.

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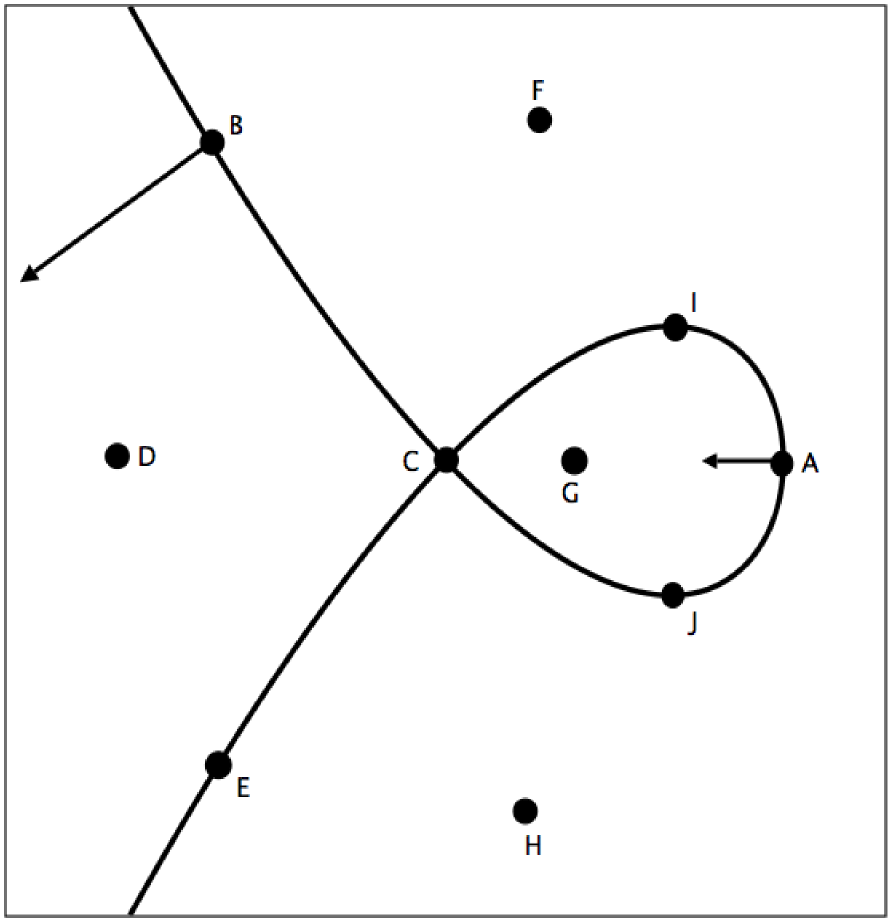
3. The curve in the following picture is the zero set of a function $f(x, y)$. We are given the gradient of f at the points A, B, as in the picture.



Answer the following questions:

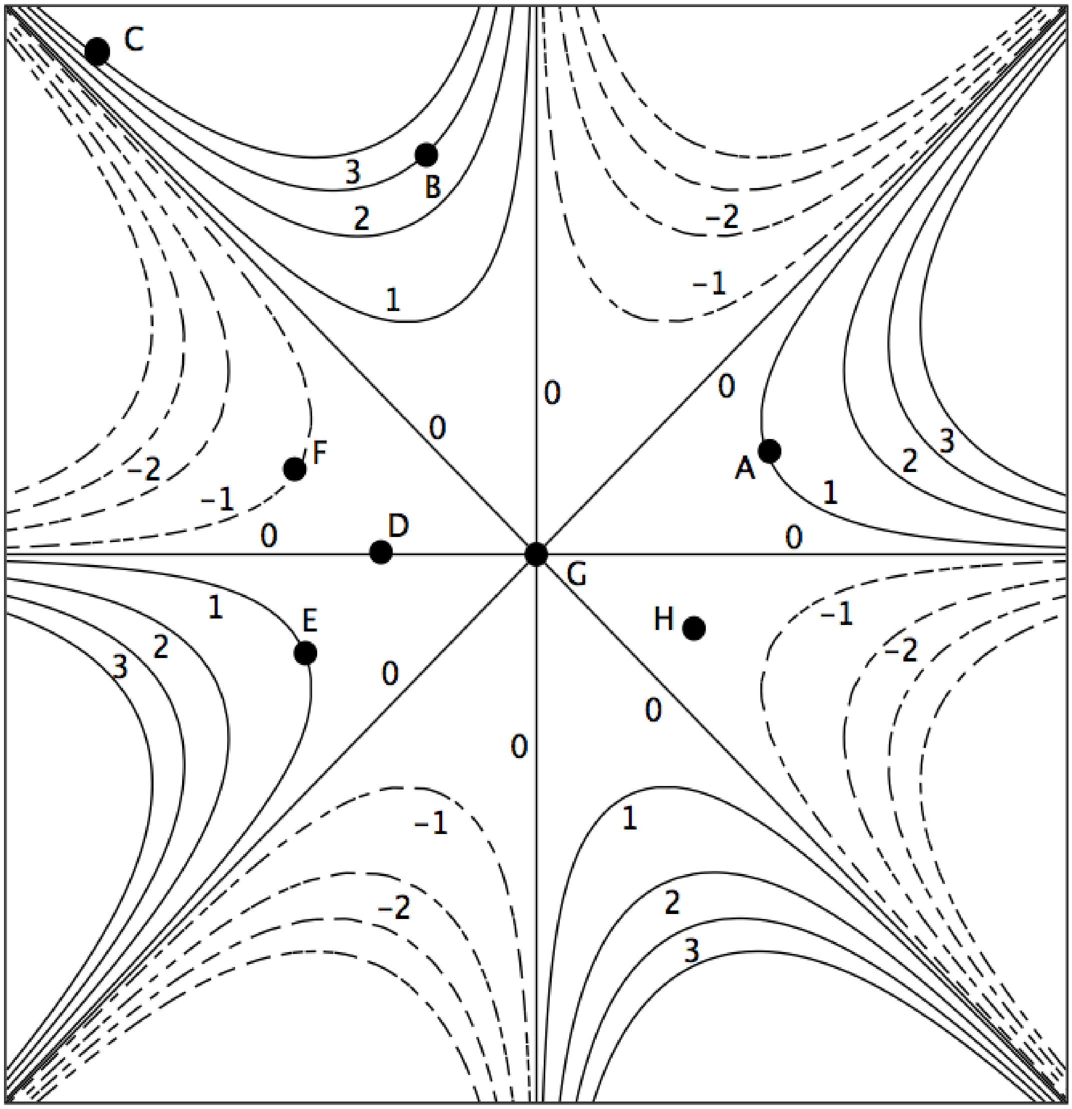
(i) What is $\vec{\nabla} f(\mathrm{C})$?

(ii) Consider all ten points A through J. At which of these is f positive? Negative? Zero?

(iii) Sketch $\vec{\nabla} f(\mathrm{E})$, $\vec{\nabla} f(\mathrm{I})$, $\vec{\nabla} f(\mathrm{J})$ in the graph.

(iv) If we were to sketch more level sets, where would you expect the level sets to be closer together, near A or near B?

4. The following picture shows an assortment of level sets for a function $f(x, y)$.



Answer the following questions:

(i) At which point in the picture is $\vec{\nabla} f(x, y) = \langle 0, 0 \rangle$?

(ii) Consider the points A, B, C. At which of these three is the magnitude of $\vec{\nabla} f$ largest? At which of these three is it smallest?

(iii) Consider the points D, E, F. At each of these points, draw the direction in which $\vec{\nabla} f$ is pointing.

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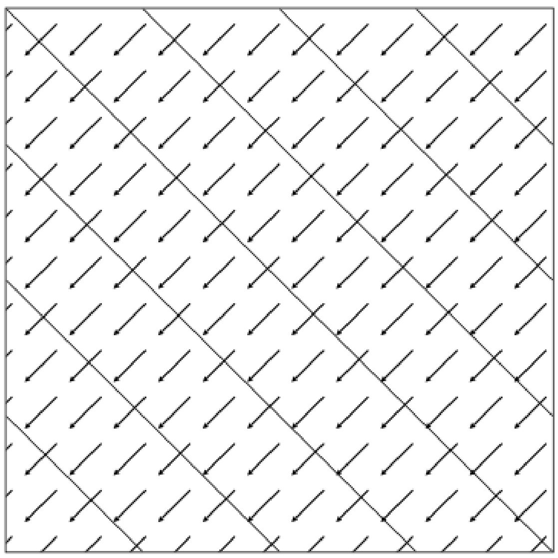
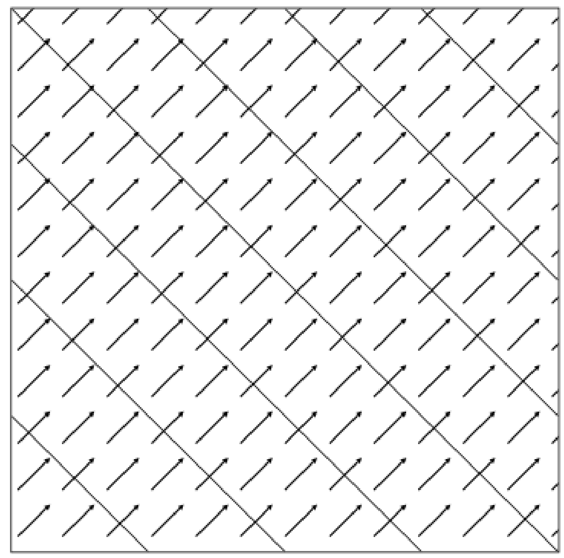
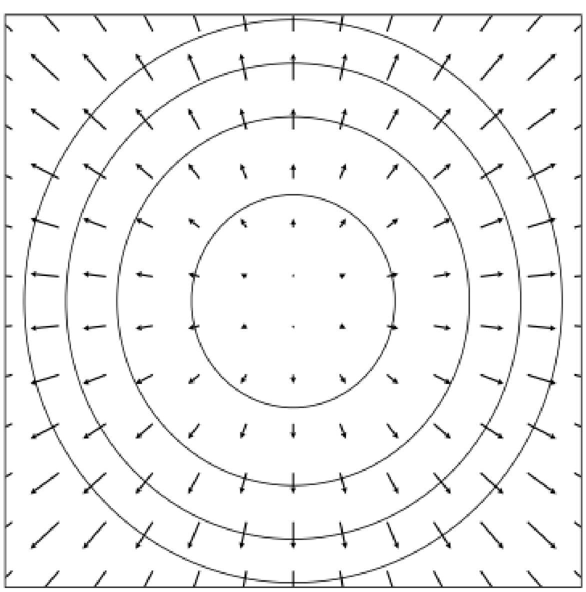
5. Consider the following functions:

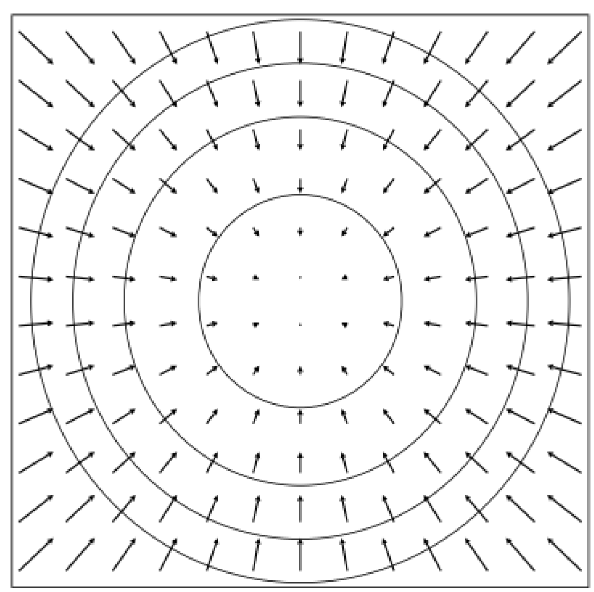
(1) $f(x, y) = x^2 - y^2$	(2) $f(x, y) = -x^2 - y^2$	(3) $f(x, y) = y^2 - x^2$
(4) $f(x, y) = x + y$	(5) $f(x, y) = x^2 + y^2$	(6) $f(x, y) = -x - y$

Match the six functions with the six pictures below. The pictures below display an assortment of level sets, but the level sets are unmarked (and the ones at negative levels are not dotted). The pictures display the gradient of the function evaluated at various sample points.









(more pictures on the back . . .)