1. For each function, find the global maximum and minimum on the given domain.
   (a) $f(x, y) = 5x - 3y$ on $x^2 + y^2 = 1$
   (b) $f(x, y) = 4x^2 + 10y^2$ on $x^2 + y^2 \leq 4$.
   (c) $f(x, y) = x^2 + 4y^2 - 2x^2y + 4$ on $\{(x, y)| - 1 \leq x \leq 1, -1 \leq y \leq 1\}$.
   (d) $f(x, y) = x^4 - y^2(1 + x^2)$ on $x^2 + y^2 \leq 16$.
   (e) $f(x, y, z) = xyz$ on $x + y + z = 1$, if $x \geq 0, y \geq 0, z \geq 0$.
   (f) $f(x, y, z) = x + y + 2z$ on $x^2 + y^2 + z^2 \leq 3$.
   (g) $f(x, y, z) = x^2 - y^2$ on $x^2 + 2y^2 + 3z^2 \leq 1$.

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2. Find the dimensions of the box with the largest volume if the total surface area is 64 cm^2.

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3. (a) Use Lagrange multipliers to show that $f(x, y, z) = z^2$ has only one critical point on the surface $x^2 + y^2 - z = 0$.
   (b) Show that the one critical point is a minimum.
   (c) Sketch the surface. Why did Lagrange multipliers not find a maximum of f on the surface?

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4. A consumer has \$600 to spend on two commodities, the first of which costs \$20 per unit and the second \$30 per unit. Suppose the utility derived by the consumer from x units of the first commodity and y units of the second commodity is given by the $U(x, y) = 10x^{0.6}y^{0.4}$. How many units of each commodity should the consumer buy to maximize utility?