1. Find the work done by the force

$$\vec{F}(x,y) = \left\langle \cos(x^6) + xy^2 - 3y\,,\; 5x + x^2y + e^y \right\rangle$$

in moving a particle along each of the following curves C:

(a) C: The triangle with vertices $(0,0)$, $(1,0)$, $(3,2)$, oriented CCW.

(b) $C = C_1 \cup C_2 \cup C_3 \cup C_4$ where

C_1 : the line segment from $(1,0)$ to $(2,0)$,
C_2 : the upper semicircle $x^2 + y^2 = 4$ (CCW),
C_3 : the line segment from $(-2,0)$ to $(-1,0)$,
C_4 : the upper semicircle $x^2 + y^2 = 1$ (clockwise).

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2. Find the integral

$$\int_C \left(\frac{x^2y^2}{2} + e^{x^5}\right) dx - y^3x\,dy,$$

where C is the path from $(1,0)$ to $(-1,0)$ along the upper half of the unit circle, followed by the straight line segment path from $(-1,0)$ to $(1,0)$.

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3. Let C be a simple, closed, piecewise smooth curve in the xy-plane and let R be the region enclosed by C. Show the following:

$$\begin{aligned} \text{Area}(R) &= \frac{1}{2}\int_C x\,dy - y\,dx, \\ \text{Area}(R) &= \int_C x\,dy, \\ \text{Area}(R) &= \int_C -y\,dx. \end{aligned}$$

Hint: for each of these, start with the right-hand side, think of it as a work (or flux) integral, and apply Green's Theorem to arrive at the left-hand side!

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4. Use any of the equations from the previous problem to compute the area enclosed by the following curves:

(a) The ellipse $x^2 + 4y^2 = 36$.

(b) $\vec{r}(t) = \left\langle t^2, \frac{t^3}{3} - t \right\rangle, \quad -\sqrt{3} \le t \le \sqrt{3}.$
