1. Evaluate the integral $\iint_R x + y\,dA$ where R is the region inside the triangle with vertices $(0,0)$, $(2,0)$ and $(0,1)$.

2. A cylinder of solid metal is given by the region in space bounded by $x^2 + y^2 = 25$ and the planes $z = 0$ and $z = 4$. The density function of the cylinder if $\rho(x,y,z) = e^{x^2+y^2}$. What is the mass of the cylinder?

3. Let C be the arc of the parabola $y = 2 - x - x^2$ between the points $(-2,0)$ and $(1,0)$, and let $\mathbf{F} = 2xe^{x^2-1}\cos(y)\mathbf{i} - e^{x^2-1}\sin(y)\mathbf{j}$ be a vector field. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{s}$.

4. Let C denote the circumference $(x-2)^2 + (y-2)^2 = 1$ traversed counterclockwise. Evaluate the integral
$$\oint_C (x^6 + 3y)\,dx + (2x - e^y)\,dy.$$

5. Let S be the portion of the cone $z = 1 - \sqrt{x^2 + y^2}$ lying above the xy-plane. We orient S by an upward normal $\mathbf{n}$. Given a vector field $\mathbf{F} = y\mathbf{i} + \sin(z^2)\mathbf{j} + \cos(x^2)\mathbf{k}$, evaluate the surface integral
$$\iint_S \operatorname{curl}(\mathbf{F}) \cdot \mathbf{n}\,d\sigma$$

6. Let S be the sphere $x^2 + y^2 + z^2 = 4$ oriented by the outward normal $\mathbf{n} = \frac{1}{2}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$ and let $\mathbf{F} = (xy + x)\mathbf{i} + (y - y^2)\mathbf{j} + (yz + z)\mathbf{k}$ be a vector field. Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\,d\sigma$.

7. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{s}$ in which C is the curve $\gamma(t) = \begin{pmatrix} t \\ t^2 \\ t^3 \end{pmatrix}$ for $0 \le t \le 1$ and $\mathbf{F}$ is the vector field $\begin{pmatrix} e^y \\ xe^y \\ (z+1)e^z \end{pmatrix}$.

8. Evaluate the line integral

$$\int_C (y + e^{\sqrt{x}})\, dx + (2x - \cos(y^2))\, dy,$$

where C is the boundary of the region enclosed by the curves $y = x^2$ and $x = y^2$, and C is oriented counterclockwise.

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9. Consider the surface S formed by the upper half of the ellipsoid $x^2 + y^2 + 6z^2 = 1$, and write C for the circle $x^2 + y^2 = 1$ where S cuts the xy-plane. We use the outer normal (upwards pointing) to orient S so that C is traversed counterclockwise in the xy-plane when viewed from above. Let $\mathbf{F} = (\sin(xz) + \sqrt{x})\mathbf{i} + ((3+z)x - e^y)\mathbf{j} + (x^2 - y^3 - z^5)\mathbf{k}$. Compute the surface integral $\int_S \nabla \times \mathbf{F} \cdot \mathbf{n}\, d\sigma$.

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10. Write S for the part of the surface $z = x^2 + y^2$ over the disk $x^2 + y^2 \leq 1$ in the plane. We orient S so that its normal $\mathbf{n}$ points downward. If the vector field $\mathbf{F} = (e^y + x)\mathbf{i} + (e^{\sin z} + \sin(x))\mathbf{j} + (-z + xy)\mathbf{k}$, then compute the surface integral $\iint \mathbf{F} \cdot \mathbf{n}\, d\sigma$.

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11. Consider the vector field

$$\mathbf{F} = \left(-\frac{z^2}{5} - z + \pi y e^{\sin x} \cos x\right)\mathbf{i} + (\pi e^{\sin x} - x)\mathbf{j} - \frac{2xz}{5}\mathbf{k}$$

and the curve C given by

$$\gamma(t) = \begin{pmatrix} 2\cos t \\ 2\sin t \\ 0 \end{pmatrix}$$

for $-\pi/2 \leq t \leq \pi/2$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{s}$.

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12. In the following, $\mathbf{F}$ is any vector field in $\mathbb{R}^3$ and f is any function in 3 dimensions. You may assume $\mathbf{F}$ and f have continuous derivatives. For each problem, state whether the given identity is true or false.

   (a) $\operatorname{div}(\nabla f) = 0$

   (b) $\operatorname{curl}(\nabla f) = \mathbf{0}$

   (c) $\operatorname{div}(\operatorname{curl} \mathbf{F}) = 0$

   (d) $\operatorname{curl}(\operatorname{curl} \mathbf{F}) = \mathbf{0}$

   (e) $\nabla(\operatorname{div} \mathbf{F}) = \mathbf{0}$

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13. Consider the function $f(x, y, z) = e^{(\sin x \cos y)} \left(z + \frac{\pi}{2}\right)$, and let C be the curve $\gamma(t) = \begin{pmatrix} t\cos^2(2t) \\ t\sin(t) \\ t \end{pmatrix}$ for $0 \le t \le \pi/2$. Compute the integral $\displaystyle\int_C \frac{\partial f}{\partial x}\, dx + \frac{\partial f}{\partial y}\, dy + \frac{\partial f}{\partial z}\, dz$.

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14. Let S be the closed surface in 3-space formed by the cone $x^2 + y^2 - z^2 = 0$, $1 \le z \le 2$, the disk $x^2 + y^2 \le 4$ in the plane $z = 2$, and the disk $x^2 + y^2 \le 1$ in the plane $z = 1$. Define the vector field $\mathbf{F}(x, y, z) = xy^2\mathbf{i} + x^2y\mathbf{j} + \sin x\mathbf{k}$ and let $\mathbf{n}$ be the outward pointing unit normal vector to A. Compute the surface integral $\displaystyle\int_S \mathbf{F} \cdot \mathbf{n}\, d\sigma$.

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15. Let C be the curve that is the intersection of the plane $x + y + z = 1$ and the cylinder $x^2 + y^2 = 9$, oriented counter-clockwise as viewed from above. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{s}$, where $\mathbf{F} = \begin{pmatrix} -yx^2 \\ y^2 z \\ z^2 \end{pmatrix}$.

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16. Let $\mathbf{F}(x, y) = \begin{pmatrix} y^3 \\ 3xy \end{pmatrix}$ be a vector field in the plane and let C be the closed curve consisting of four piecewise smooth pieces where C_1 is the top half of the circle $x^2 + y^2 = 4$, C_3 is the top half of the circle $x^2 + y^2 = 1$, and C_2 and C_4 are line segments of unit length along the x-axis which connect the two semicircles. Orient this curve in a counterclockwise orientation. Evaluate the integral $\displaystyle\oint_C \mathbf{F} \cdot d\mathbf{s}$.

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17. True or False?

    (a) If S is any closed surface, then $\displaystyle\int_S \nabla \times \mathbf{F}\, d\sigma = 0$.

    (b) If $\mathbf{F} = \nabla f$, then $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{s} = 0$ for all closed curves C.

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18. Find the area of the portion of the plane $z = 10 + 2x + 3y$ over the disk $x^2 + y^2 \le 1$.

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19. Find the flux of the field $\mathbf{F} = x\mathbf{i} + y\mathbf{j} - z\mathbf{k}$ across the cylindrical surface $x^2 + z^2 = 1$, $0 \le y \le 3$, with outward-pointing normal.

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20. The curl of $\mathbf{F}$ is $\nabla \times \mathbf{F} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. What is the work done by $\mathbf{F}$ along the oriented square path from $(1, -1, 3)$ to $(2, -1, 3)$ to $(2, 0, 3)$ to $(1, 0, 3)$ and back to $(1, -1, 3)$?
