

1. (a) $f(x) = \frac{1}{2} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin((2n-1)\pi x/L)}{2n-1}$
(b) $f(x) = \frac{3L}{4} + \sum_{n=1}^{\infty} \left(\frac{2L \cos((2n-1)\pi x/L)}{(2n-1)^2 \pi^2} + \frac{(-1)^{n+1} L \sin(n\pi x/L)}{n\pi} \right)$
2. $f(x) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin((2n-1)\pi x)}{2n-1}$
3. $f(x) = \sum_{n=1}^{\infty} \frac{2}{n\pi} \left(-\cos n\pi + \frac{2}{n\pi} \sin \frac{n\pi}{2} \right) \sin \frac{n\pi x}{2}$
4. (a) $X'' - \lambda x X = 0, \quad T' + \lambda t T = 0$
(b) $X'' - \lambda(X' + X) = 0, \quad T' + \lambda T = 0$
5. (a) $u(x, t) = \frac{100}{\pi} \sum_{n=1}^{\infty} \frac{1 - \cos n\pi}{n} e^{-n^2 \pi^2 t / 1600} \sin \frac{n\pi x}{40}$
(b) $u(x, t) = \frac{160}{\pi^2} \sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{n^2} e^{-n^2 \pi^2 t / 1600} \sin \frac{n\pi x}{40}$
6. (a) $aw_{xx} - bw_t + (c - b\delta)w = 0$
(b) $\delta = c/b$, if $b \neq 0$
7. $X'' + \mu^2 X = 0, \quad Y'' + (\lambda^2 - \mu^2)Y = 0, \quad T' + \alpha^2 \lambda^2 T = 0$