

The following exercises are from the corresponding sections of the UC Berkeley custom edition of Lay, Nagle, Saff, & Snider, Linear Algebra and Differential Equations. Note that the section numbers and problem numbers ARE NOT the same as in Lay, Linear Algebra.

**Exercises 1.3:** 14, 15

**Exercises 1.5:** 6, 7, 18, 24, 31

**Exercises 1.7:** 5, 11, 27, 33-38, 40

**Exercises 1.8:** 3, 12, 17, 25, 34

**Exercises 1.9:** 7, 11, 19, 27, 33

**Optional Additional Problems:** (you do not need to hand these in, but I suggest you try them.)

1. Prove that if a set  $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$  is linearly dependent, then for some  $i$  between 1 and  $n$ ,  $\vec{v}_i$  is a linear combination of  $\vec{v}_1, \dots, \vec{v}_{i-1}$ . (We discussed this in class; here I want you to try to write down all the details.)
2. Let  $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$  be a linear transformation. Suppose  $\{\vec{u}, \vec{v}\}$  is a linearly independent set, but  $\{T(\vec{u}), T(\vec{v})\}$  is a linearly dependent set. Show that the equation  $T(\vec{x}) = \vec{0}$  has a non-trivial solution.
3. Explain why a set  $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$  in  $\mathbb{R}^5$  must be linearly independent when  $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$  is linearly independent and  $\vec{v}_4$  is not in  $\text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ .
4. Suppose  $\vec{v}_1, \vec{v}_2, \vec{v}_3$  are distinct points on one line in  $\mathbb{R}^3$  (the line need not pass through the origin). Show that  $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$  is linearly dependent.