The following exercises are from the corresponding sections of the UC Berkeley custom edition of Lay, Nagle, Saff, & Snider, Linear Algebra and Differential Equations. Note that the section numbers and problem numbers ARE NOT the same as in Lay, Linear Algebra.

**Exercises 4.3:** 15, 21, 23, 31, 33, 34

**Exercises 4.4:** 5, 11, 21, 23, 24, 27, 32

**Exercises 4.5:** 13, 21, 25, 27, 29, 32

**Exercises 4.6:** 11, 24, 27, 30, 31, 34

**Additional Problems:**

1. Let $T : V \to W$ be a linear transformation of vector spaces. Prove that the range of $T$ is a subspace of $W$.

2. Find a set of vectors that span $\text{Nul}(A)$, where

$$A = \begin{pmatrix} 1 & 6 & 4 & -11 & 2 \\ 2 & 2 & -2 & 9 & 3 \\ -1 & 2 & 4 & -10 & -5 \end{pmatrix}.$$

3. Let $\mathcal{B} = \{\vec{v}_1, \ldots, \vec{v}_k\}$ be a spanning subset of a vector space $V$. Suppose no proper subspace of $\mathcal{B}$ spans $V$. Prove that $\mathcal{B}$ is linearly independent (and so $\mathcal{B}$ is a basis for $V$). Hint: it is easier to prove the contrapositive, that if $\mathcal{B}$ is dependent, then it has a proper subset that spans $V$.

4. If $W_1$ and $W_2$ are subspaces of a vector space $V$, define their *sum* to be $W_1 + W_2 = \{w_1 + w_2 \mid w_1 \in W_1, w_2 \in W_2\}$. Prove that

   (a) the sum $W_1 + W_2$ is a subspace of $V$, and

   (b) the intersection $W_1 \cap W_2$ is a subspace of $V$.

5. Prove that if $V$ is finite dimensional and $W \subseteq V$ is a subspace, then

   (a) $\dim(W) \leq \dim(V)$

   (b) if $\dim(W) = \dim(V)$ then $W = V$.

6. Show that for any $m \times n$ matrix $A$ and $n \times l$ matrix $B$, the matrix equation $AB = 0$ is equivalent to $\text{Col}(B) \subseteq \text{Nul}(A)$.