Meyer Solutions

① $4x^2+y^2+9z^2=14$, $(-1,1,1)$

Let $f(x,y,z)=4x^2+y^2+9z^2$

$\vec{n}=\vec{\nabla}f(x,y,z)=\langle 8x, 2y, 18z\rangle$, because $\vec{\nabla}f$ is normal to the level set.

$\vec{n}=\vec{\nabla}f(-1,1,1)=\langle -8,2,18\rangle$

$\vec{n}\cdot\vec{AX}=0$ $(A=(-1,1,1))$

$\langle -8,2,18\rangle\cdot\langle x+1, y-1, z-1\rangle=0$

$\boxed{-8(x+1)+2(y-1)+18(z-1)=0}$ Tangent Plane.

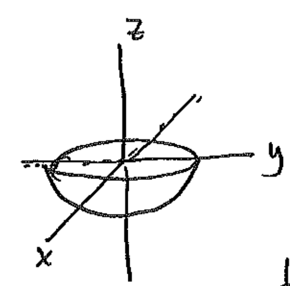
$\|\vec{n}\|=\sqrt{64+4+18^2}$ ← too big.

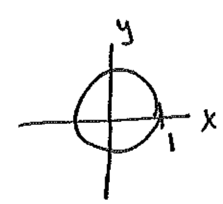
$\vec{n}=2\langle -4,1,9\rangle$. let $\vec{m}=\langle -4,1,9\rangle$, which is still normal to the tan. plane.

$\|\vec{m}\|=\sqrt{16+1+81}=\sqrt{98}=7\sqrt{2}$

$\boxed{\text{unit normal: }\left\langle \frac{-4}{7\sqrt{2}}, \frac{1}{7\sqrt{2}}, \frac{9}{7\sqrt{2}}\right\rangle}$

③ $f(x,y,z)=e^{x^2+y^2+z^2}$, $x^2+y^2+z^2\le 1$, $z\le 0$





(a) Cylindrical

$r^2+z^2=1 \rightarrow z=-\sqrt{1-r^2}$ (b/c $z\le 0$)

$$\boxed{\int_0^{2\pi}\int_0^1\int_{-\sqrt{1-r^2}}^{0} r\cdot e^{r^2+z^2}\,dz\,dr\,d\theta}$$

(b) Spherical

$0\le\rho\le 1$, $\underbrace{\pi/2\le\phi\le\pi}_{z\le 0}$

$$\boxed{\int_0^{2\pi}\int_{\pi/2}^{\pi}\int_0^1 e^{\rho^2}\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta}$$

④ (a) $\vec{F}(x,y,z) = \langle 9x^8e^y,\ x^9e^y+1,\ 1\rangle$

$$\int 9x^8e^y\,dx = x^9e^y + C(y,z)$$

$$\int x^9e^y+1\,dy = x^9e^y + y + D(x,z)$$

$$\int 1\,dz = z + E(x,y)$$

So, $\vec{F}(x,y,z) = \vec{\nabla} f(x,y,z)$, where $f(x,y,z) = x^9e^y + y + z$,

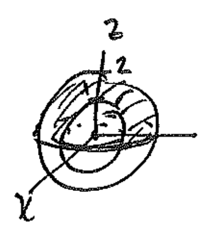
so yes, $\vec{F}$ is conservative.

(b) $(0,0,0)$ to $(1,2,1)$. By the FTC for line integrals,

$$\int_C 9x^8e^y\,dx + (x^9e^y+1)\,dy + dz = f(1,2,1) - f(0,0,0)$$

$$= 1e^2 + 2 + 1 = \boxed{e^2+3}$$

⑤



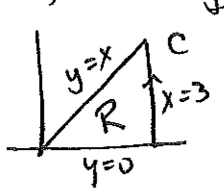
$$\int_0^{2\pi}\int_0^{\pi/2}\int_1^2 \frac{1}{\rho}\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/2}\int_1^2 \rho\sin\phi\,d\rho\,d\phi\,d\theta = \int_0^{2\pi}\int_0^{\pi/2} \frac{\rho^2}{2}\Big|_1^2 \sin\phi\,d\phi\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/2} \left(2-\tfrac{1}{2}\right)\sin\phi\,d\phi\,d\theta = \frac{3}{2}\int_0^{2\pi} -\cos\phi\Big|_0^{\pi/2} d\theta$$

$$= \frac{3}{2}\int_0^{2\pi}(0+1)\,d\theta = \frac{3}{2}\cdot 2\pi = \boxed{3\pi}$$

⑥ (a) $\vec{F} = \langle \underbrace{y^2-x^2}_{P},\ \underbrace{x^2+y^2}_{Q}\rangle$. By Green's Thm,



$Q_x = 2x$

$P_y = 2y$

$$\int_C \vec{F}\cdot d\vec{x} = \iint_R (2x-2y)\,dA$$

$$= \int_0^3\int_0^x (2x-2y)\,dy\,dx = \int_0^3 2xy - y^2\Big|_0^x dx$$

$$= \int_0^3 2x^2 - x^2\,dx = \int_0^3 x^2\,dx = \frac{x^3}{3}\Big|_0^3 = \boxed{9}$$