MATH 222(1,2,4) Fall 2015
Quiz 10 RM Solutions

Please inform your TA if you find any errors in the quiz solutions.

1. (*4 points*)
Let $\vec{\mathbf{a}} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{\mathbf{b}} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ and $\vec{\mathbf{c}} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Which of the following expressions are nonsense? Evaluate the sensible ones.

1. $\vec{\mathbf{a}} + \vec{\mathbf{c}}$
2. $\vec{\mathbf{a}} \cdot \vec{\mathbf{c}}$
3. $\vec{\mathbf{a}}\vec{\mathbf{b}}$
4. $\vec{\mathbf{a}} - 2\vec{\mathbf{b}}$

Solution:

1. Nonsense
2. Nonsense
3. Nonsense
4. $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - 2\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 5 \end{pmatrix}$

2. (*6 points*)
Let $\vec{\mathbf{a}} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ and $\vec{\mathbf{b}} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$. Find $\vec{\mathbf{a}}^{/\!/}$ and $\vec{\mathbf{a}}^{\perp}$ so that $\vec{\mathbf{a}} = \vec{\mathbf{a}}^{/\!/} + \vec{\mathbf{a}}^{\perp}$, where $\vec{\mathbf{a}}^{/\!/}$ is parallel to $\vec{\mathbf{b}}$ and $\vec{\mathbf{a}}^{\perp}$ is perpendicular to $\vec{\mathbf{b}}$.
Solution:

$$\begin{aligned} \vec{\mathbf{a}}^{/\!/} &= \left(\vec{\mathbf{a}} \cdot \frac{\vec{\mathbf{b}}}{\|\vec{\mathbf{b}}\|}\right) \frac{\vec{\mathbf{b}}}{\|\vec{\mathbf{b}}\|} \\ &= \left(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}\right) \frac{\vec{\mathbf{b}}}{\|\vec{\mathbf{b}}\|^2} \end{aligned}$$

Observe that $\|\vec{\mathbf{b}}\| = \sqrt{1+1} = \sqrt{2}$ and $\left(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}\right) = 2+1 = 3$. Then

$$\vec{\mathbf{a}}^{/\!/} = \begin{pmatrix} \frac{3}{2} \\ \frac{3}{2} \\ 0 \end{pmatrix}$$

and

$$\begin{aligned} \vec{\mathbf{a}}^{\perp} &= \vec{\mathbf{a}} - \vec{\mathbf{a}}^{/\!/} \\ &= \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} \frac{3}{2} \\ \frac{3}{2} \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{pmatrix} \end{aligned}$$