MATH 222(1,2,4) Fall 2015

Quiz 3 RM Solutions

Please inform your TA if you find any errors in the quiz solutions.

1. (*5 points*) Compute the antiderivative

$$\int \frac{x\left(x^3-2x^2+x\right)+3}{x^3-2x^2+x}dx$$

Solution: First write the integrand in reduced form.

$$\frac{x\left(x^3-2x^2+x\right)+3}{x^3-2x^2+x} = x + \frac{3}{x^3-2x^2+x}$$

Now apply partial fractions to the second summand.

$$\frac{3}{x^3-2x^2+x} = \frac{1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

Solving for A, B and C:

$$\begin{aligned}\frac{3}{x(x-1)^2} &= \frac{A(x-1)^2+Bx(x-1)+Cx}{x(x-1)^2}\\ 3 &= A(x^2-2x+1)+B(x^2-x)+Cx = (A+B)x^2+(-2A-B+C)x+A\end{aligned}$$

This implies the three equations

$$\begin{aligned}0 &= A+B\\ 0 &= -2A-B+C\\ 3 &= A\end{aligned}$$

Solving this system of equations we get $B=-3$ and $C=3$. Thus

$$\begin{aligned}\int\left(x+\frac{2}{x^3-2x^2+x}\right)dx &= \frac{1}{2}x^2+\int\left(\frac{3}{x}+\frac{-3}{x-1}+\frac{3}{(x-1)^2}\right)dx\\ &= \frac{1}{2}x^2+3\left(\ln|x|-\ln|x-1|-\frac{1}{x-1}\right)+C\end{aligned}$$

2. (*5 points*) Compute the antiderivative

$$\int\sqrt{7-x^2}dx \tag{1}$$

Solution:

$$\begin{aligned}
\int \sqrt{7-x^2}dx &= \int \left(\sqrt{7-7\sin^2(\theta)}\right)\sqrt{7}\cos(\theta)d\theta && x=\sqrt{7}\sin(\theta)\\
&= \int \sqrt{7}\cos(\theta)\sqrt{7(1-\sin^2(\theta))}d\theta && dx=\sqrt{7}\cos(\theta)d\theta\\
&= 7\int \cos^2(\theta)d\theta\\
&= 7\int \frac{1+\cos(2\theta)}{2}d\theta\\
&= 7\left(\frac{1}{2}\theta+\frac{1}{4}\sin(2\theta)\right)+C\\
&= \frac{7}{2}\arcsin(\frac{x}{\sqrt{7}})+\frac{7}{4}\sin(2\arcsin(\frac{x}{\sqrt{7}}))+C\\
&= \frac{7}{2}\arcsin(\frac{x}{\sqrt{7}})+\frac{7}{2}\sin(\arcsin(\frac{x}{\sqrt{7}}))\cos(\arcsin(\frac{x}{\sqrt{7}}))+C\\
&= \frac{7}{2}\arcsin(\frac{x}{\sqrt{7}})+\frac{\sqrt{7}}{2}x\sqrt{1-\left(\frac{x}{\sqrt{7}}\right)^2}+C
\end{aligned}$$