MATH 222(1,2,4) Fall 2015
Quiz 8 RM Solutions

Please inform your TA if you find any errors in the quiz solutions.

1. (*4 points*)

For each of the following, circle true or false:

$(x+x^3)^3 = o(x^3)$	True	False
$\cos(x^2) - 1 = o(x^3)$	True	False
$e^x - 1 = o(x)$	True	False
$\sin(x) - x = o(x^2)$	True	False

Solution:

1. False
2. True
3. False
4. True

2. (*6 points*)

Suppose that $y(x)$ is a solution to

$$\begin{aligned} 0 &= y''(x) + y(x) + 3x \\ y(0) = 2 \qquad & y'(0) = 3. \end{aligned}$$

Compute the degree three Taylor polynomial of $y(x)$ around zero.
Solution: Write

$$\begin{aligned} y(x) &= y(0) + y'(0)x + \frac{y''(0)}{2}x^2 + \frac{y'''(0)}{3!}x^3 + o(x^3) \\ &= a_0 + a_1 x + a_2 x^2 + a_3 x^3 + o(x^3) \\ &= a_0 + a_1 x + o(x) \\ y''(x) &= 2a_2 + 6a_3 x + o(x). \end{aligned}$$

Substituting in, we have

$$\begin{aligned} 0 &= y''(x) + y(x) + 3x \\ &= (2a_2 + 6a_3 x + o(x)) + (a_0 + a_1 x + o(x)) + 3x \\ &= (2a_2 + a_0) + (6a_3 + a_1 + 3)\, x + o(x). \end{aligned}$$

Equating coefficients, we see that

$$0 = 2a_2 + a_0 \tag{1}$$
$$0 = 6a_3 + a_1 + 3. \tag{2}$$

We are given that $a_0 = y(0) = 2$ and $a_1 = y'(0) = 3$. Plugging $a_0 = 2$ into (1), we see that $a_2 = -1$ and plugging $a_1 = 3$ into (2), we see that $a_3 = -1$. We conclude that the degree three Taylor polynomial of $y(x)$ is given by $f(0)+f'(0)x+\frac{f''(0)}{2}x^2+\frac{f'''(x)}{3!}x^3 = a_0+a_1x+a_2x^2+a_3x^3 = 2 + 3x - x^2 - x^3$.