Review Sheet Solutions

① $A(0,2,5)$, $B(8,-4,1)$, $C(2,9,3)$

$\vec{AB} = \langle 8,-6,-4\rangle$
$\vec{AC} = \langle 2,7,-2\rangle$

$$\vec{n} = \vec{AB}\times\vec{AC} = \begin{vmatrix} \vec{i} & 8 & 2 \\ \vec{j} & -6 & 7 \\ \vec{k} & -4 & -2 \end{vmatrix} = \vec{i}(12+28) - \vec{j}(-16+8) + \vec{k}(56+12) = \langle 40, +8, 68\rangle$$

$\vec{n}\cdot\vec{AX} = 0$

$\langle 40,8,68\rangle \cdot \langle x, y-2, z-5\rangle = 0$

$40x + 8y - 16 + 66z - 340 = 0$

$$\boxed{40x + 8y + 66z = 356.}$$

*Note: Since only the direction of $\vec{n}$ matters, not the length, you could simplify $\vec{n}$ to be $\langle 10, 2, 17\rangle$, & get $10x + 2y + 17z = 89$.

② area of $\Delta ABC = \frac{1}{2}$ area of parallelogram spanned by $\vec{AB}$ & $\vec{AC}$,

so area of $\Delta ABC = \left|\frac{1}{2}(\vec{AB}\times\vec{AC})\right|$

$= \left|\frac{1}{2}\langle 8,-6,-4\rangle \times \langle 2,7,-2\rangle\right|$

$= \frac{1}{2}\|\langle 40, 8, 68\rangle\|$

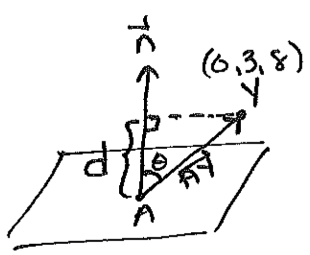
$= \frac{1}{2}\sqrt{1600+64+4642}$

$= \boxed{\frac{1}{2}\sqrt{6288}}$

$$\begin{vmatrix} \vec{i} & 8 & 2 \\ \vec{j} & -6 & 7 \\ \vec{k} & 4 & -2 \end{vmatrix} = \langle 40,8,68\rangle$$

from #1

③



d = distance from Y to plane.

$d = \|\vec{AY}\|\cdot\cos\theta$

$$= \frac{\vec{n}\cdot\vec{AY}}{\|\vec{n}\|} = \frac{\langle 40,8,68\rangle\cdot\langle 0,1,3\rangle}{\sqrt{40^2+8^2+66^2}} = \frac{8+204}{\sqrt{40^2+8^2+66^2}} = \boxed{\frac{212}{\sqrt{40^2+8^2+66^2}}}$$

Since $\vec{n}\cdot\vec{AY} > 0$, θ is acute, so Y lies on the same side of the plane as the normal vector $\vec{n}$.

④ Let $\vec{m}$ be the normal vector for this plane. Since the ∡ between the planes is 90°, $\vec{m} \cdot \vec{n} = 0$, so

$$\langle m_1, m_2, m_3 \rangle \cdot \langle 40, 8, 68 \rangle = 0$$

$$40m_1 + 8m_2 + 68m_3 = 0$$

Choose $m_3 = 0$, $m_2 = 5$, & $m_1 = -1$ (not the only choice)

So $\vec{m} = \langle 0, 5, -1 \rangle$

So the equation is:

$$\vec{m} \cdot \langle x-0, y-0, z-0 \rangle = 0$$

$$\langle 0, 5, -1 \rangle \cdot \langle x, y, z \rangle = 0$$

$$\boxed{5y - z = 0}$$

⑤ The line passes through $Y(0,3,8)$ & is in the direction of $\vec{n} = \langle 40, 8, 68 \rangle$ (because it's normal to the plane), so

$$\boxed{\vec{l}(t) = \langle 0,3,8 \rangle + t\langle 40, 8, 68 \rangle.}$$

⑥ $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \vec{a} \times \vec{a} + \vec{b} \times \vec{a} - \vec{a} \times \vec{b} - \vec{b} \times \vec{b}$

$= \vec{0} + \vec{b} \times \vec{a} - (-\vec{b} \times \vec{a}) - \vec{0}$

$= \boxed{2\vec{b} \times \vec{a}}$

⑦ $\vec{x}(t) = \langle \cos t, \sin t, 4t^2 \rangle$

$\vec{v}(t) = \vec{x}'(t) = \langle -\sin t, \cos t, 8t \rangle$

$\vec{v}(\pi/2) = \langle -\sin \frac{\pi}{2}, \cos \frac{\pi}{2}, 8 \cdot \pi/2 \rangle = \boxed{\langle -1, 0, 4\pi \rangle}$

$\vec{a}(t) = \vec{v}'(t) = \langle -\cos t, -\sin t, 8 \rangle$

$\boxed{\vec{a}(\pi/2) = \langle 0, -1, 8 \rangle}$

Speed $= \| \vec{v}(\pi/2) \| = \sqrt{(-1)^2 + 0^2 + (4\pi)^2} = \boxed{\sqrt{1 + 16\pi^2}}$

⑧ $\vec{x}(t) = \langle \cos t, \sin t, 4t^2 \rangle \rightsquigarrow \vec{x}(t_0) = \langle \cos t_0, \sin t_0, 4t_0^2 \rangle$

$\vec{v}(t) = \langle -\sin t, \cos t, 8t \rangle \rightsquigarrow \vec{v}(t_0) = \langle -\sin t_0, \cos t_0, 8t_0 \rangle$

Tangent line: $\vec{\ell}(s) = \vec{x}(t_0) + s\,\vec{v}(t_0)$

$$\boxed{\vec{\ell}(s) = \langle \cos t_0, \sin t_0, 4t_0^2 \rangle + s\langle -\sin t_0, \cos t_0, 8t_0 \rangle}$$

⑨ $f(x,y) = \sin x - y$

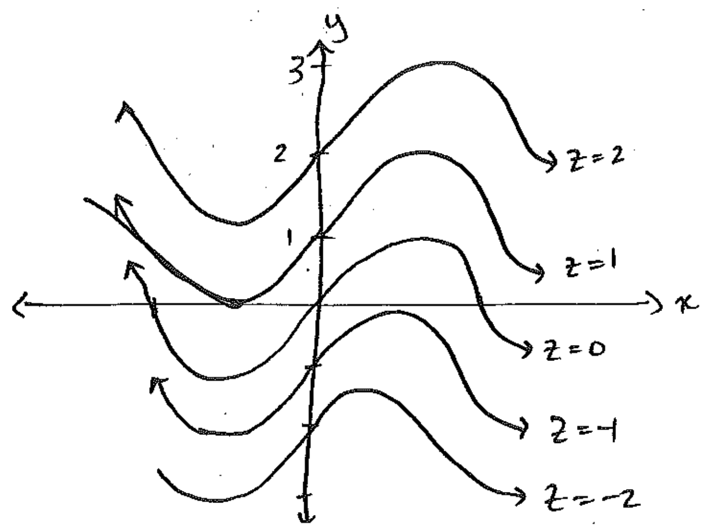
$z=0$: $y = \sin x$

$z=1$: $y = \sin x - 1$

$z=2$: $y = \sin x - 2$

$z=-1$: $y = \sin x + 1$

$z=-2$: $y = \sin x + 2$



⑩ $f(x,y) = -2x^2 + 4xy - 9y^2$

$A = -2$
$B = 4$
$C = -9$

$4AC - B^2 = 4(-2)(-9) - (4)^2 = 72 - 16 > 0$

$\Rightarrow$ definite, & $A<0$, so $\boxed{\text{negative definite}}$

$\boxed{\text{parabaloid}}$

⑪ $f(x,y) = -2x^2 + 4xy - 9y^2$

$= -2(x^2 - 2xy + y^2) - 9y^2 + 2y^2$

$$\boxed{f(x,y) = -2(x-y)^2 - 7y^2}$$

(12) $f(x,y,z) = \dfrac{x\sin(yz)}{x^2 z} + z = \dfrac{\sin(yz)}{xz} + z$

$$\boxed{f_x = -\frac{\sin(yz)}{x^2 z}}$$

$$\boxed{f_y = \frac{\cos(yz)\cdot z}{xz} = \frac{\cos(yz)}{x}}$$

$$\boxed{f_z = \frac{(xz)\cos(yz)\cdot y - x\sin(yz)}{x^2 z^2} + 1}$$

(13) $f(x,y) = x^2 + xy + y^2$ at $(2,1)$

$f_x = 2x+y$, $f_y = x+2y$

$f_x(2,1) = 4+1 = 5$, $f_y(2,1) = 2+2 = 4$

So, $\boxed{\Delta f = 5\Delta x + 4\Delta y}$