$\det A_n = (-1)\det\begin{pmatrix} 1 & 0 \\ 0 & A_{n-2} \end{pmatrix}$, by performing cofactor expansion down the first column of A_n.

Note that $\det\begin{pmatrix} 1 & 0 \\ 0 & A_{n-2} \end{pmatrix} = \det A_{n-2}$

If n is odd, then the base case is $A_1 = (0)$, and $\det A_1 = 0$. Thus $\det A_3 = (-1)\det\begin{pmatrix} 1 & 0 \\ 0 & A_1 \end{pmatrix} = (-1)\det A_1 = 0$. Continuing in this manner, we see that $\boxed{\det A_n = 0}$.

If n is even, then the base case is $A_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, and $\det A_2 = -1$. Thus $\det A_4 = (-1)\det\begin{pmatrix} 1 & 0 \\ 0 & A_2 \end{pmatrix} = 1$

and $\det A_6 = (-1)\det\begin{pmatrix} 1 & 0 \\ 0 & A_4 \end{pmatrix} = -1$.

Continuing in this manner, we see that the determinants alternate sign, but always have absolute value 1. Thus $\boxed{\det A_n = (-1)^{n/2}}$.

Note: There are many solutions to this problem; if you want to know if your solution is correct, I'd be happy to discuss it with you.