1. For each integral, change the order of integration and then compute the integral.

   (a) $\int_0^2 \int_{\sqrt{x}}^2 y\,dy\,dx$

   (b) $\int_0^1 \int_{-y}^{y} x^2\,dx\,dy$

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2. If D is the cylinder of radius 2 and height 3 defined by $x^2 + y^2 \leq 4$ and $0 \leq z \leq 3$, compute the following integral:

$$\iiint_D (x^2 + y^2 - z^2)\,dx\,dy\,dz$$

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3. Let $f(x, y) = 3xy - x^2 - y$ and D be the domain defined by $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Find the maximum and minimum of f on the closed domain D.

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4. Find the equation of the tangent plane to the surface $xy - yz + e^{xz} = 3$ at the point where $x = 0$ and $y = -1$. Then approximate the value of z after increasing x by .1 and decreasing y by .1.

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5. Find the total mass of a solid D bounded by the cone $z = \sqrt{x^2 + y^2} - 1$ between the xy-plane and the plane $z = 1$ with mass density $\mu(x, y, z) = \frac{1}{z+1}$.

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6. Let $\rho = \sqrt{x^2 + y^2 + z^2}$. Show that $\dfrac{\partial \rho}{\partial x} = \dfrac{x}{\rho}$. Then let $\mathbf{F}(x, y, z) = \rho^2 x\mathbf{i} + \rho^2 y\mathbf{j} + \rho^2 z\mathbf{k}$ and find $\nabla \cdot \mathbf{F}$ in terms of ρ.

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7. Let $f(x, y) = xe^{x-2y}$ and approximate the value $f(1.99, 1.02)$.

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8. Suppose we are given a function $f(x, y)$. Define $g(u, v) = f(u \ln v, u + v)$. In the following, your answer may contain the variables u and v as well as the partial derivative of f with respect to x and/or y.

   (a) Compute $\dfrac{\partial g}{\partial u}$.

   (b) Compute $\dfrac{\partial^2 g}{\partial u \partial v}$.

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9. Find and classify all critical points of the function $f(x, y) = x^2 - 3y^2 + y^3$.

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10. Optimize the function xy^2 if $x^2 + y^2 = 3$.

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11. Compute the volume under the graph of $z = \frac{1}{(x+y)^2}$ above the rectangle $3 \le x \le 6$ and $0 \le y \le 2$.

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12. Let R the the region given by $x \ge 0$, $y \ge x$, $x^2 + y^2 \le 4$, and $0 \le z \le x^2 + y^2$. Use cylindrical coordinates to find the volume of R.

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13. Let R be the region inside the sphere $x^2 + y^2 + z^2 \le 9$ given by $x \ge 0$, $z \ge 0$ and $y \ge x$. Use spherical coordinates to write the integral

$$I = \iiint_R x \, dV.$$

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14. Find the mass of the solid region inside the sphere of radius 2 and outside the sphere of radius 1, both centered at the origin, and above the plane $z = 0$, if the density is $\delta(x, y, z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$.

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15. Let S be the part of the surface $z = 1 - y^2$ which lies above the rectangle $0 \le x \le 1$ and $0 \le y \le 1$. Find $\displaystyle\int_S xy \, d\sigma$.

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16. Let C be the curve $\gamma(t) = e^t \cos t\mathbf{i} + e^t \sin t\mathbf{j} + e^t\mathbf{k}$, with $-1 \le t \le 1$. Find the length of C, the unit tangent vector, the unit normal vector, the acceleration vector, and the curvature of C.

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17. Find the points on the surface $x^2 + 1 = 2yz$ closest to the origin.

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18. Consider the function $z = f(x, y)$ be defined near $(x, y) = (1, 1)$ by $x - y + e^{xz-x} + z = 2$, with $f(1, 1) = 1$.

    (a) Find the equation of the tangent plane to the graph $z = f(x, y)$ at $(x, y, z) = (1, 1, 1)$.

    (b) Find a direction in which $f(x, y)$ decreases most rapidly at $(1, 1)$.

    (c) Determine if there is a direction v in which the directional derivative at $(1, 1)$ is $1/4$.

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19. Find a point on the surface $z = x^2 - 2xy - y^2 - 8x + 4y$ where the tangent plane is horizontal.