Topics on the Exam

1. Vectors
   - Adding/subtracting vectors, algebraically and geometrically
   - Multiplying a vector by a scalar, algebraically and geometrically
   - Dot product, both formulas
   - Parallel vectors, perpendicular vectors
   - Cross product, how to find it and the geometric interpretation
   - Magnitude and finding a unit vector
   - Equation of a plane (using a normal vector)
2. Parametric Equations
   - Parametric equation for a line, circle, and helix
   - Arc length, arc length function, arc length parameter
   - Velociy and speed
   - Acceleration, and how to decompose it into linear and angular acceleration
   - Unit tangent, unit normal, unit binormal
   - Be sure you can sketch the above vectors on a picture of the curve
   - Curvature, both how to find it and how to tell from a picture whether the curvature is large or small
   - Equation of a tangent line to a curve (in 2 or 3 dimensions)
3. Function of Several Variables
   - Domain
   - Level sets
   - Sketching the graph of a surface
   - Polar coordinates
   - Equations of basic surfaces (paraboloid, saddle, parabolic cylinder, cone, double cone)
   - Partial derivatives
   - Derivatives in the direction of a path (i.e., $(f \circ \gamma)'(t)$.
   - Equation of a tangent plane to a surface

- Linear approximation

Practice Problems

1. Find a unit vector in the direction of $\begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$.

2. Are $\mathbf{u} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$ parallel? Perpendicular?

3. Consider the points $A(0, 2, 4)$, $B(3, -1, 6)$ and $C(2, 5, 4)$.
   (a) Write the equation of the plane containing A, B, and C.
   (b) Find the area of triangle ABC.
   (c) Write a parametric equation for the line passing through A and B.

4. Find the angle between the vectors $\mathbf{u} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$. (Leave your answer in terms of an inverse trig function)

5. Write a parametric equation for a particle traveling around a circle centered at the origin, with radius 2, if the particle travels clockwise and makes one full rotation from time 0 to π.

6. For the circle above, find its velocity, speed, unit tangent, and unit normal vectors.

7. If a particle is moving with a constant velocity, what can you say about its velocity and acceleration vectors?

8. Find the curvature of the helix $\mathbf{x}(t) = \begin{pmatrix} \cos t \\ \sin t \\ 2t \end{pmatrix}$. How does this helix compare to the standard helix $\begin{pmatrix} \cos t \\ \sin t \\ t \end{pmatrix}$?

9. Find the equation of the tangent line to the helix $\mathbf{x}(t)$ in the previous problem at the point $(1, 0, 4\pi)$. Where does this line intersect the xy-plane?

10. Sketch the domain of $f(x, y) = \sqrt{x^2 - y} + \ln(x - y)$.

11. Draw at least 4 level sets of $f(x, y) = \frac{1}{xy}$.

12. Sketch the graphs of $f(x,y) = x^2 + y^2$, $g(x,y) = x^2 - y^2$, $h(x,y) = \sqrt{x^2 + y^2}$, and $k(x,y) = x^2$. [These should all be familiar functions.]

13. How is the graph of $f(x,y) = x^2 + y^2 + 4$ related to the graph of $g(x,y) = x^2 + y^2$.

14. Find f_x and f_y of $f(x,y) = x^2 y - \sin(x^2 + y)$.

15. Write the equation of the tangent plane to the surface $xyz = 1$ at the point $(2,1)$.

16. Approximate the value of $f(x,y) = \frac{1}{xy}$ at the point $(1.9, 1.01)$ and at the point $(0.1, 1.99)$.

17. If $f(x,y) = \frac{1}{xy}$, and $\gamma(t) = \begin{pmatrix} t^2 \\ t \end{pmatrix}$, find $(f \circ \gamma)'(1)$. What does this value tell you about the surface $f(x,y)$?