Topics on the Exam

1. Functions of Several Variables
   - Directional derivative: how to calculate it and what it means
   - Gradient
     - Perpendicular to level sets
     - Points in the direction of maximum increase of the function
   - Multivariable chain rule
2. Optimization
   - Local maxima/minima
   - Global maxima/minima on a closed bounded region
     - Use the gradient to deal with the interior (if there is one)
     - Parametrize the boundary
     - Use substitution to deal with the boundary
     - Use Lagrange multipliers to deal with the boundary
   - Word problems about optimization
3. Integration
   - Double integrals
     - Cartesian coordinates
     - Polar coordinates
     - Sketching the domain and switching the bounds
   - Triple Integrals
     - Cartesian coordinates
     - Cylindrical coordinates
     - Spherical coordinates
     - Sketching the domain and switching the bounds

Practice Problems

1. Find the derivative of $f(x, y) = x^2 + y^2$ in the direction of the vector $\mathbf{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ at the point (2,1,5). What does this value tell you about the surface f?
2. Find the gradient of $f(x, y) = 2x\cos(xy) - e^{x^2+y^2}$.

3. Suppose that a differentiable function $f(x, y)$ satisfies

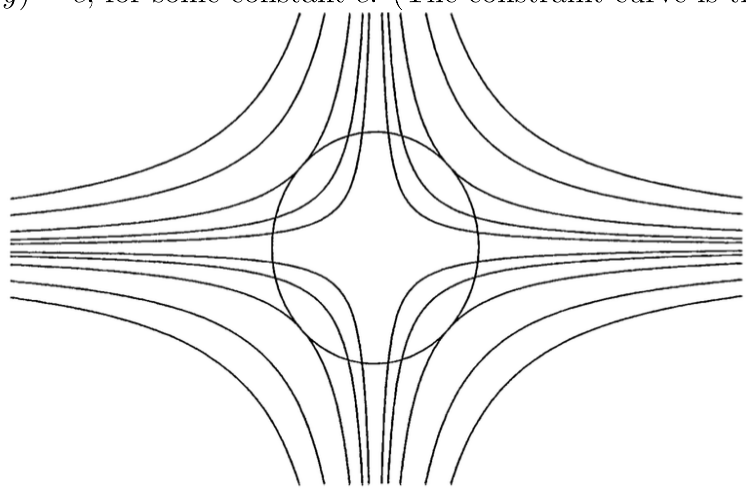
$$\nabla f(1,2) = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \qquad \nabla f(6,2) = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Furthermore suppose

$$x = 2s + 2t \qquad y = st$$

Define $h(s,t) = f(x,y)$ and find $\frac{\partial h}{\partial s}(1,2)$.

4. The image below shows the level sets of a function $z = f(x, y)$ and a constraint curve, $g(x, y) = c$, for some constant c. (The constraint curve is the oval in the



middle.)

   (a) On the graph, mark the approximate locations of all critical points (a.k.a potential extrema) of $f(x, y)$ subject to the constrain $g(x, y) = c$.

   (b) If $f(x, y) = xy$ and the constraint function is $g(x, y) = x^2 + y^2$, find all global extrema of $f(x, y)$ subject to the constrain $g(x, y) = 2$.

5. Find the direction of maximal increase of $f(x, y) = 2x^3y + 5\ln(y)x^2$ at the point $(1, 1)$. In what directions could you move so that the value of f does not change? Give your answer as a vector.

6. Let $f(x, y) = x^2 + y^2$. Where on $x^2 + y^2 = 1$ is $\nabla \mathbf{f}$ parallel to $\langle 2, -3 \rangle$?

7. Graph the zero set of $f(x, y) = x^3 - x^2y$. From the zero set, what are the possible critical points? Can you classify them using only the zero set. (You can check your answers with the 2nd derivative test)

8. Consider the surface $f(x, y) = x^2 + y^2$. For which values of x do the level sets of f define y as an implicit function of x? For these values, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. For which values of y do the level sets of f define x as an implicit function of y? For these values, find $\frac{\mathrm{d}x}{\mathrm{d}y}$.

9. (a) If $g(x,y) = f(u,v,z)$, where $u = 3x+5$, $v = 2x+y^2$, and $z = xy$, find all first-order partial derivatives of g. You may leave your answer in terms of (derivatives of) f.

   (b) If $g(x,y) = f(x^2+2xy+y^2)$, where $f(t)$ is a function of one variable find all first-order partial derivatives of g. You may leave your answer in terms of (derivatives of) f.

10. Find and classify all critical points of $f(x,y) = x^3 + 2x^2y^2$. Are any of these local extrema also global extrema? Why/why not?

11. Find the global extrema of $f(x,y) = x^2+3xy+y^2$ on the rectangle $\{(x,y)|-2 \leq x \leq 2, 0 \leq y \leq 3\}$.

12. Find $\iint\limits_R (4-y^2)\,\mathrm{d}A$ where $R = \{(x,y)|0 \leq x \leq 3, 0 \leq y \leq 2\}$.

13. Find the volume under $f(x,y) = 1$ over the region D bounded by $y = x$ and $x = y^2$.

14. Find the volume under $f(x,y) = 6x$ over the region D bounded by $x^2+y^2 = 4$, $x \geq 0$.

15. Reverse the limits of integration (you do not need to evaluate): $\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2}\,\mathrm{d}x\mathrm{d}y$.

16. Finish worksheet 18 (triple integral practice). The solutions are already posted on my website.

17. Find the shortest distance from a point on the plane $x+2y+2z = 3$ to the origin.

18. Find the maximum and minimum values of the function $f(x,y,z) = x$ over the curve of intersection of the plane $z = x+y$ and the ellipsoid $x^2+2y^2+2z^2 = 8$.

19. Find the dimensions of the box with largest volume if the total surface area is 64 cm^2.