1. Let $z = xe^{xy}$, $x = st$, and $y = t^2 - s^2$. Find z_s and z_t using the chain rule.

2. Suppose that a differentiable function f satisfies
$$\frac{\partial f}{\partial x}(x, y) = xy, \quad \frac{\partial f}{\partial y}(x, y) = x^2/2.$$
Furthermore suppose
$$x = ts^2, \quad y = s/t.$$
Find $\dfrac{\partial f}{\partial t}$ and $\dfrac{\partial f}{\partial s}$ using the chain rule.

3. Suppose that a differentiable function $f(x, y, z)$ satisfies
$$\vec{\nabla} f(8, 4, 12) = \begin{pmatrix} A \\ B \\ C \end{pmatrix}, \quad \vec{\nabla} f(12, 27, 3) = \begin{pmatrix} D \\ E \\ F \end{pmatrix}, \quad \vec{\nabla} f(12, 32, 2) = \begin{pmatrix} G \\ H \\ I \end{pmatrix}, \quad \vec{\nabla} f(12, 27, 6) = \begin{pmatrix} J \\ K \\ L \end{pmatrix}.$$
Furthermore suppose
$$x = u + v, \quad y = uv, \quad z = u/v.$$
Define $h(u, v) = f(x, y, z)$. Find $\dfrac{\partial h}{\partial u}(8, 4)$ and $\dfrac{\partial h}{\partial v}(8, 4)$. Also find $\dfrac{\partial h}{\partial u}(9, 3)$ and $\dfrac{\partial h}{\partial v}(9, 3)$.

4. Let $f(x, y)$ be differentiable, and consider the polar variables r, θ.
   (a) Find $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial \theta}$ in terms of r, θ, f_x, f_y.
   (b) Use part (a) to simplify the following two expressions:
$$\cos\theta \frac{\partial f}{\partial r} - \frac{1}{r}\sin\theta \frac{\partial f}{\partial \theta} \qquad\qquad \sin\theta \frac{\partial f}{\partial r} + \frac{1}{r}\cos\theta \frac{\partial f}{\partial \theta}$$
   (c) Use what you got in part (b) to express $\|\vec{\nabla} f\|$ in terms of $r, \theta, \dfrac{\partial f}{\partial r}, \dfrac{\partial f}{\partial \theta}$.

5. Write the equation of the tangent plane to $x^2 + 2y^2 + 2z^2 = 7$ at $(\sqrt{2}, 1, 1)$.

6. Let $f(x, y, z) = \sqrt{x^2 + y^2} - z$. Describe the level surfaces for this function, and sketch the surfaces corresponding to $c = 0$, $c = 1$, and $c = -1$. You may wish to draw each level surface on a separate graph.

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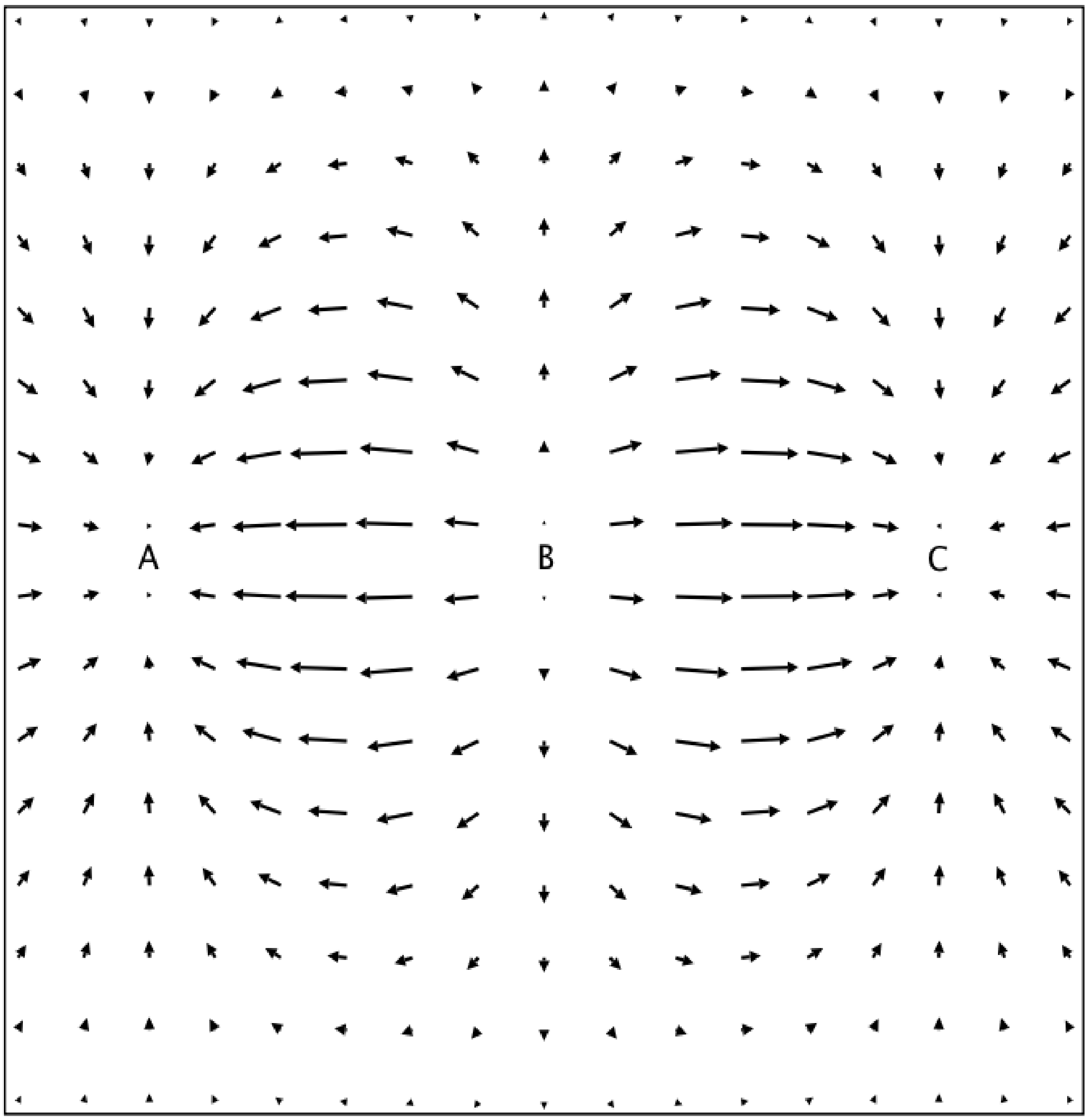
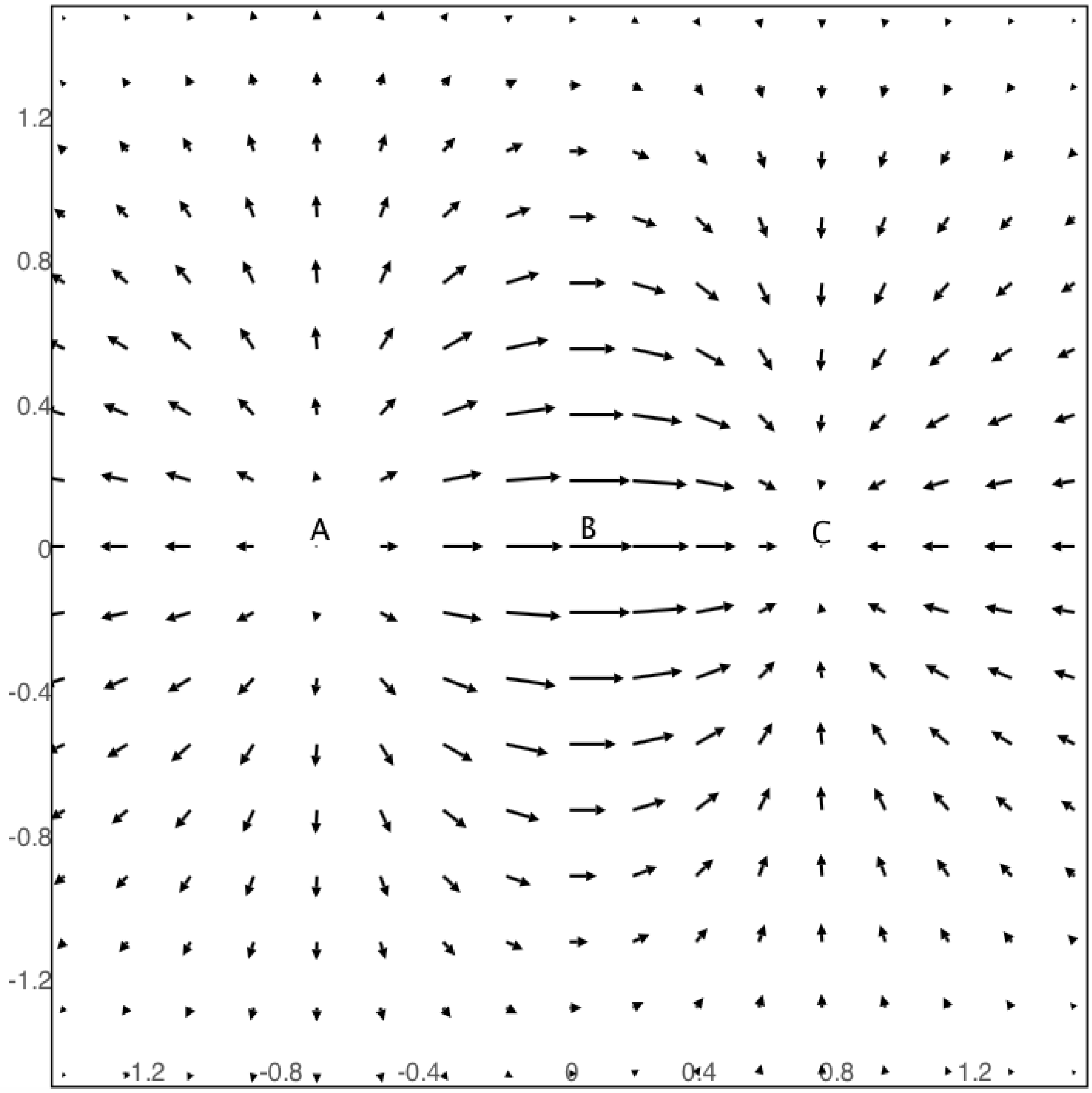
7. The picture below shows the plot of the gradient of a function at various sample points. For each of the labelled points A, B, C, determine if the function attains a local maximum, local minimum, or neither at that location.

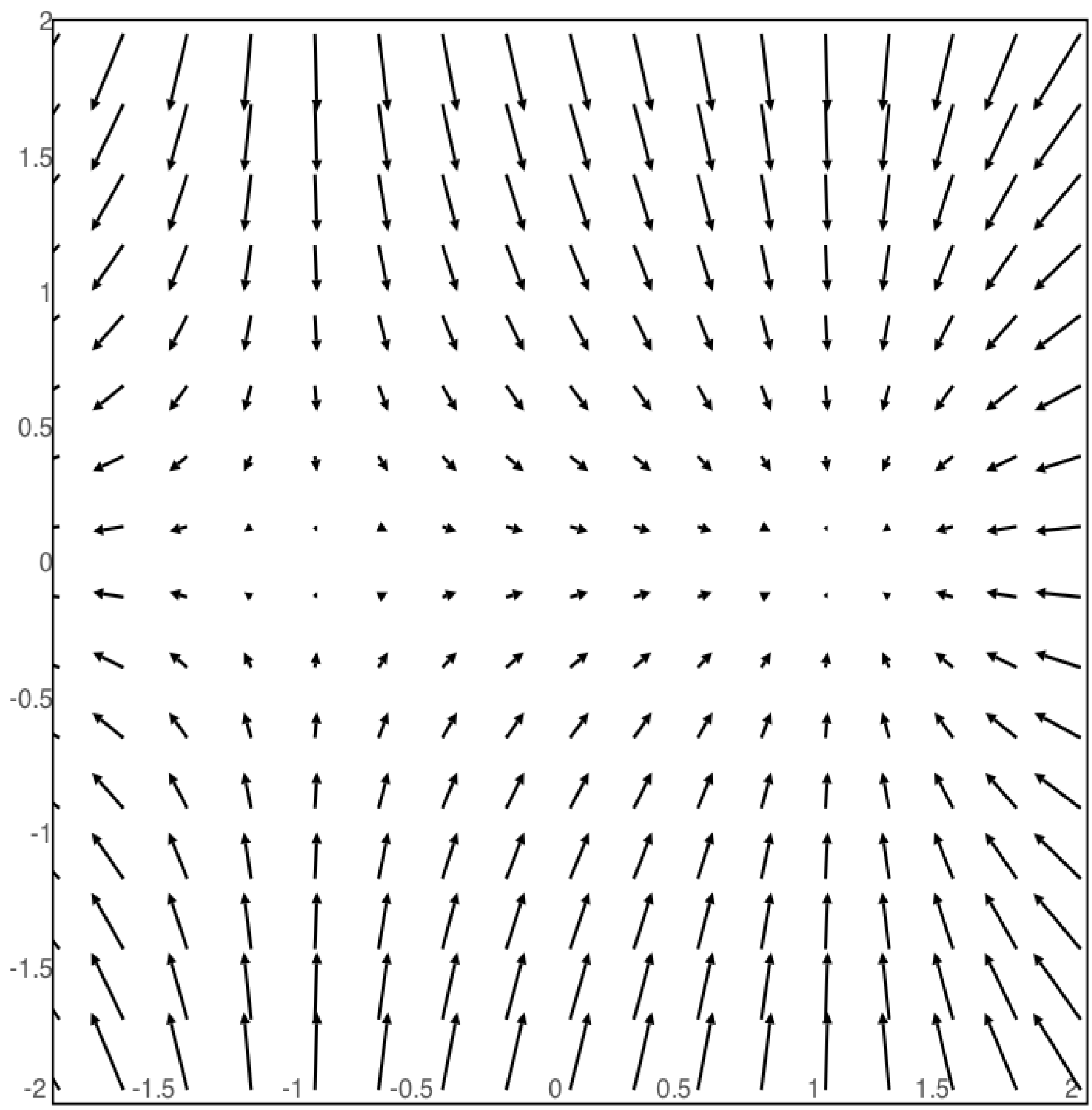


8. The picture below shows the plot of the gradient of a function at various sample points. For each of the labelled points A, B, C, determine if the function attains a local maximum, local minimum, or neither at that location.



9. The temperature at any point (x, y) in the xy-plane is given by $f(x, y)$. The picture below shows the plot of the gradient of $f(x, y)$ at various sample points.

(a) A crazy fire ant starts moving at the point $(-1/2, 2)$ and always moves in the direction of greatest increase in temperature, until it reaches the highest temperature possible. Make an approximate sketch of the path of the ant, and label where the ant stops.



(b) What would the path look like if the crazy ant started at the point $(2, -2)$ instead?