1. For each function, do the following: (i) find all critical points; (ii) determine if each critical point found is a local/global maximum/minimum or saddle point.

   (a) $f(x, y) = x^2 + 2y^2 - 2x + 8y + 11$
   (b) $f(x, y) = 2x^2 - y^2 - 16x - 2y + 37$
   (c) $f(x, y) = -3x^2 - 2y^2 - 18x + 4y - 24$
   (d) $f(x, y) = x^2 + 6xy + 11y^2 - 4y + 3$
   (e) $f(x, y) = x^2 - 10xy + 22y^2 - 12y - 8$
   (f) $f(x, y) = x^2 + y^4 - 8y^2$
   (g) $f(x, y) = xy(10 - x - 5y)$
   (h) $f(x, y) = x^2y - y^3/3$
   (i) $f(x, y) = (y - 1)(y - x^2)$
   (j) $f(x, y) = (x - 2)(x - y^2)$
   (k) $f(x, y) = (x - y)(xy - 4)$
   (l) $f(x, y) = (x^2 + y^2 - 25)^2$

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2. Consider the function $f(x, y) = \cos(x)\cos(y)$, and let's only worry about what is happening in the window $-2\pi < x < 2\pi$, $-2\pi < y < 2\pi$.

   (a) Draw the zero set of f.
   (b) Where is f positive and where is f negative?
   (c) Can you guess where the critical points will be, simply from your picture above?
   (d) From the critical points that you've identified above, which are local maxima, which are local minima, and which are saddle points?

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3. Suppose that a function $f(x, y)$ factors as $g(x, y)h(x, y)$. Prove the following fact:

   If a point (a, b) lies on the zero set of both g and h, then (a, b) is a critical point of f.

   (*Hint: use the product rule.*)

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4. Consider the function $f(x, y, z) = x^2 + y^2 - z^2$.

   (a) Find and sketch the zero set of f.

(b) Sketch the regions in space where f is positive and where f is negative.

(c) Find the critical points of f.

(d) For each critical point of f, determine if it is a local/global maximum/minimum or saddle point.

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5. Consider the two parametrized lines given by

$$\mathbf{x}_1(t) = \begin{pmatrix} t \\ t+1 \\ t \end{pmatrix}, \qquad \mathbf{x}_2(u) = \begin{pmatrix} -u \\ u \\ u \end{pmatrix}.$$

Find the shortest distance between these two lines.

(*Hint: first write down the distance between an arbitrary point on one line and an arbitrary point on the other. This should be a function of two variables. In order to minimize this, it is enough to minimize the square of this. Minimize it!*)