
Consider the vector fields $\mathbf{F} = \begin{pmatrix} 0 \\ 0 \\ -z \end{pmatrix}$ and $\mathbf{G} = \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}$, as well as the surfaces:

- Σ_1 is the portion of the cone $z = \sqrt{x^2 + y^2}$ under the plane $z = 1$.
- Σ_2 is the portion of the cylinder $x^2 + z^2 = 1$ between the planes $y = -1$ and $y = 1$.
- Σ_3 is the unit sphere $x^2 + y^2 + z^2 = 1$.

For each surface the normal vector is pointing to the "inside" of the surface.

1. By picturing the vector fields and the surfaces, for each i, decide if:

   (a) $\int_{\Sigma_i} \mathbf{F} \cdot d\sigma$ is positive, negative, or 0.

   (b) $\int_{\Sigma_i} \mathbf{G} \cdot d\sigma$ is positive, negative, or 0.

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2. Find the flux of $\mathbf{F}$ over Σ_1 and Σ_3, and the flux of $\mathbf{G}$ over Σ_2.

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3. Let Σ be the portion of the surface $3x - 3y + z = 12$ lying inside the cylinder $x^2 + y^2 = 1$, oriented with normals pointing upwards. Let $\mathbf{F}(x, y, z) = \begin{pmatrix} -x^2 \\ 0 \\ -3y^2 \end{pmatrix}$. Find the flux, $\displaystyle\int_{\Sigma} \mathbf{F} \cdot d\sigma$.
