

1. Find the outward flux of \mathbf{F} across the boundary of the region D .

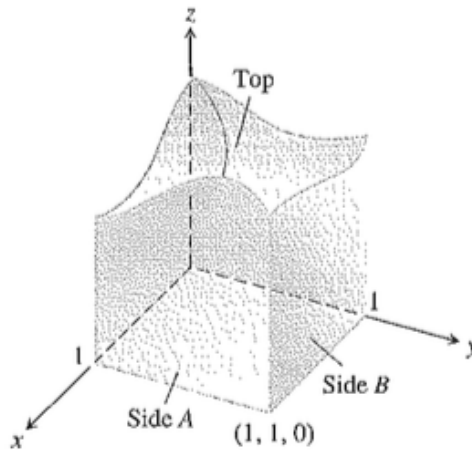
(a) $\mathbf{F} = (y - x)\mathbf{i} + (z - y)\mathbf{j} + (y - x)\mathbf{k}$, and D is the cube bounded by the planes $x = \pm 1$, $y = \pm 1$, and $z = \pm 1$.

(b) $\mathbf{F} = \begin{pmatrix} x^2 \\ y^2 \\ z^2 \end{pmatrix}$ and D is the region cut from the solid cylinder $x^2 + y^2 \leq 4$ by the planes $z = 0$ and $z = 1$.

(c) $\mathbf{F} = y\mathbf{i} + xy\mathbf{j} - z\mathbf{k}$ and D is the region inside the solid cylinder $x^2 + y^2 \leq 4$ between the plane $z = 0$ and the paraboloid $z = x^2 + y^2$.

(d) $\mathbf{F} = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$ and D is the solid sphere $x^2 + y^2 + z^2 \leq a$.

2. The base of the closed cubelike surface shown here is the unit square in the xy -plane. The four sides lie in the planes $x = 1$, $x = -1$, $y = 1$, and $y = -1$. The top is an arbitrary smooth surface whose identity is unknown. Let $\mathbf{F} = x\mathbf{i} - 2y\mathbf{j} + (z + 3)\mathbf{k}$ and suppose the outward flux of \mathbf{F} through side A is 1 and through side B is -3. Can you conclude anything about the outward flux through the top?



3. Show that the flux of the position vector field $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ outward through a smooth closed surface Σ is three times the volume of the region enclosed by the surface.

4. Among all rectangular solids defined by the inequalities $0 \leq x \leq a$, $0 \leq y \leq b$, $0 \leq z \leq 1$, find the one for which the total flux of $\mathbf{F} = (-x^2 - 4xy)\mathbf{i} - 6yz\mathbf{j} + 12z\mathbf{k}$ outward through the six sides is the greatest. What is the greatest flux?