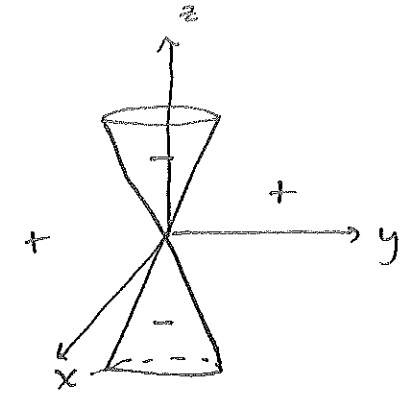
Worksheet 13 Solutions

④ $f(x,y,z) = x^2+y^2-z^2$

(a) $0 = x^2+y^2-z^2$

$z^2 = x^2+y^2$

$z = \pm\sqrt{x^2+y^2}$ (double cone)



(b) 3 regions to check:

A: Inside upper cone — A: Test $(0,0,1)$: $-$

B: Inside lower cone — B: Test $(0,0,-1)$: $-$

C: Outside double cone. — C: Test $(0,1,0)$: $+$

(c) CP's are where $\nabla f = 0$

$$\nabla f = \begin{pmatrix} 2x \\ 2y \\ -2z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{matrix} x=0 \\ y=0 \\ z=0 \end{matrix} \Rightarrow (0,0,0)$$

(d) Since there is a positive & a negative direction from $(0,0,0)$, it is a saddle point.

⑤ $x_1(t) = \begin{pmatrix} t \\ t+1 \\ t \end{pmatrix}$, $x_2(t) = \begin{pmatrix} -u \\ u \\ u \end{pmatrix}$

The distance between 2 pts in 3-space is:

$d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$. We want (x_1,y_1,z_1) to lie on $x_1(t)$ & (x_2,y_2,z_2) to lie on $x_2(t)$, so we plug in the corresponding components:

$$d = \sqrt{(t+u)^2+(t+1-u)^2+(t-u)^2}$$

Let's minimize the square of this:

$$f(t,u) = (t+u)^2+(t+1-u)^2+(t-u)^2$$

$$\nabla f = \begin{pmatrix} 2(t+u)+2(t+1-u)+2(t-u) \\ 2(t+u)-2(t+1-u)-2(t-u) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} 2t+2u+2t+2-2u+2t-2u=0 \\ 2t+2u-2t-2+2u-2t+2u=0 \end{cases} \Rightarrow \begin{cases} 6t-2u=-2 \longrightarrow u=3t+1 \\ -2t+6u=2 \end{cases}$$

$$-2t+6(3t+1)=2 \qquad u=3(-1/4)+1$$

$$16t=-4 \qquad u=1/4.$$

$$t=-1/4$$

Plugging these values into our distance formula, d, we get

$$d=\sqrt{(-1/4+1/4)^2+(-1/4+1-1/4)^2+(-1/4-1/4)^2}$$

$$=\sqrt{(1/2)^2+(-1/2)^2}$$

$$=\sqrt{1/2}$$

$$=\boxed{1/\sqrt{2}}$$