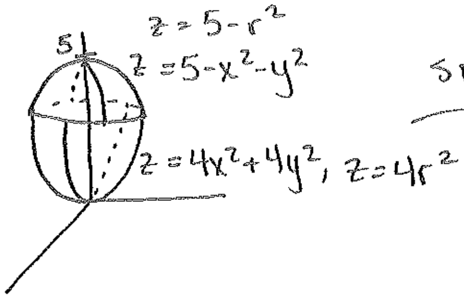
Worksheet 18 Solutions

(1)



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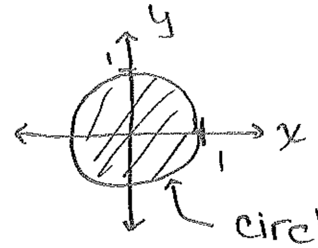


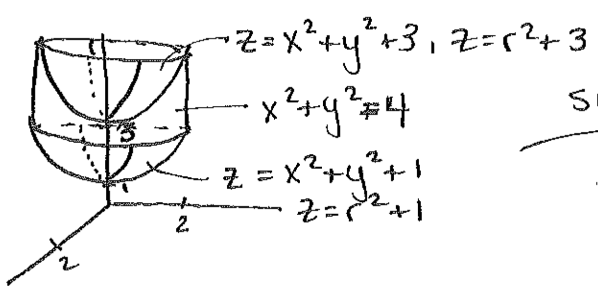
circle is the intersection of the surfaces:

$$5-x^2-y^2=4x^2+4y^2$$
$$1=x^2+y^2$$

Cartesian: $$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{4x^2+4y^2}^{5-x^2-y^2} dz\,dy\,dx$$

Cylindrical: $$\int_0^{2\pi}\int_0^1\int_{4r^2}^{5-r^2} r\,dz\,dr\,d\theta$$

(2)



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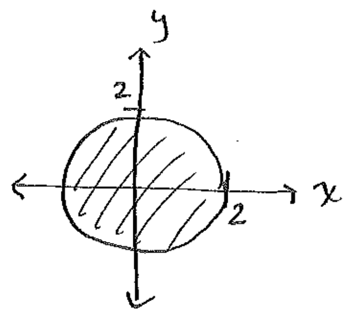
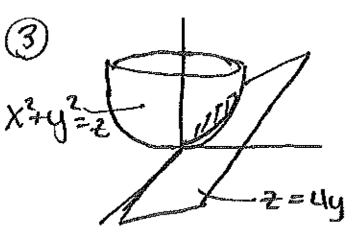


Cartesian: $$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2+1}^{x^2+y^2+3} dz\,dy\,dx$$

Cylindrical: $$\int_0^{2\pi}\int_0^2\int_{r^2+1}^{r^2+3} r\,dz\,dr\,d\theta$$

(3)



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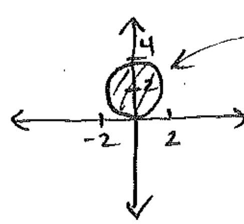


the circle is the intersection of the surfaces:

$$x^2+y^2=4y$$
$$x^2+y^2-4y=0$$
$$x^2+(y-2)^2=4$$

Cartesian: $$\int_0^4\int_{-\sqrt{4-(y-2)^2}}^{\sqrt{4-(y-2)^2}}\int_{x^2+y^2}^{4y} dz\,dx\,dy$$

Cylindrical: $x^2+y^2=4y$

$$r^2=4r\sin\theta$$

$$r^2-4r\sin\theta=0$$

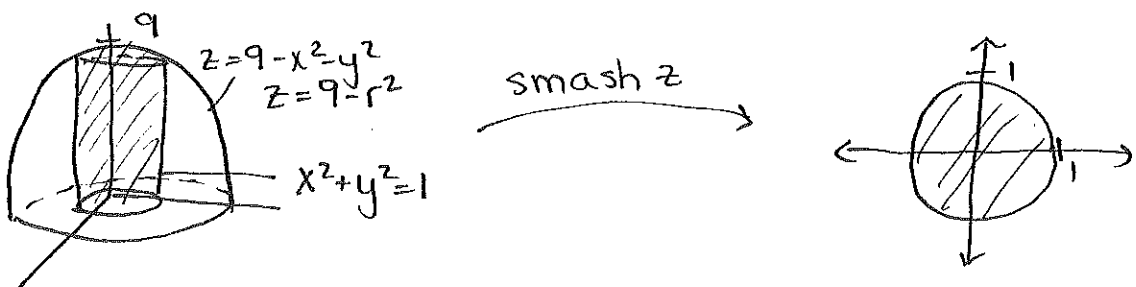
$$r(r-4\sin\theta)=0$$

$r=0$ $\quad$ $r=4\sin\theta$

Clearly not the eqn of the circle $\qquad$ eqn of the circle

$$\int_0^{\pi}\int_0^{4\sin\theta}\int_{r^2}^{4\sin\theta\cdot r} r\,dz\,dr\,d\theta$$

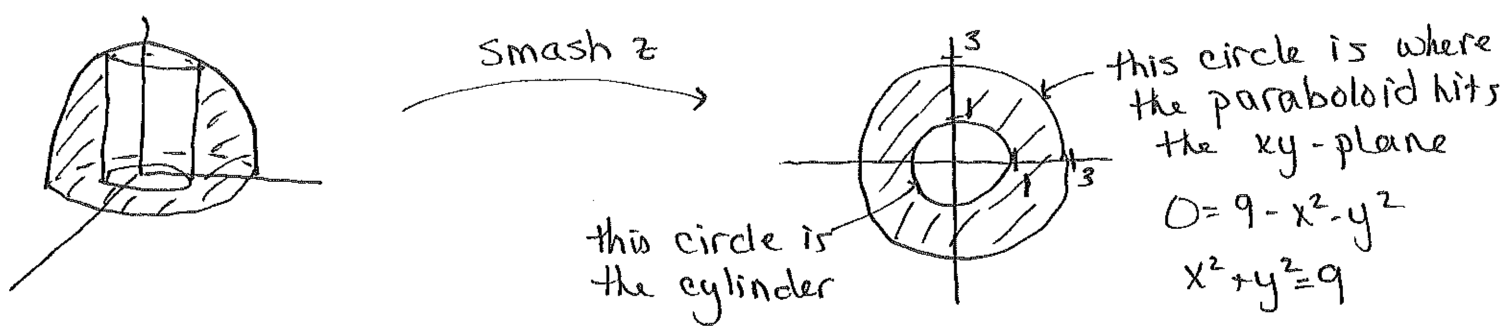
4



Cartesian: $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_0^{9-x^2-y^2} dz\,dy\,dx$

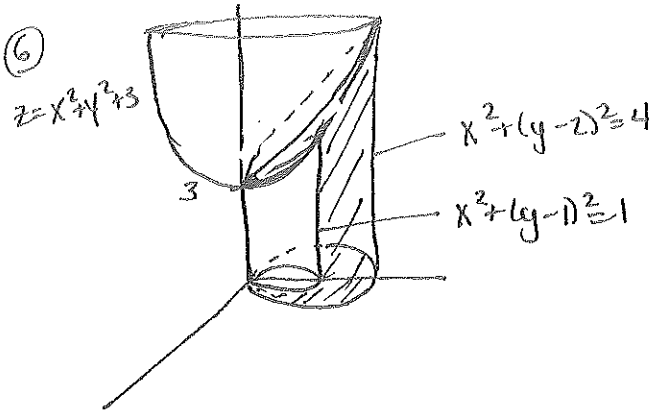
Cylindrical: $\displaystyle\int_0^{2\pi}\int_0^{1}\int_0^{9-r^2} r\,dz\,dr\,d\theta$

5



Cylindrical: $\displaystyle\int_0^{2\pi}\int_1^{3}\int_0^{9-r^2} r\,dz\,dr\,d\theta$

⑥



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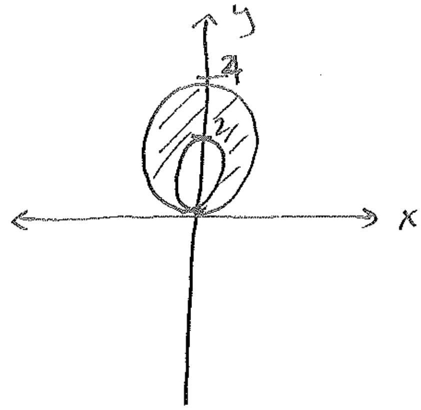


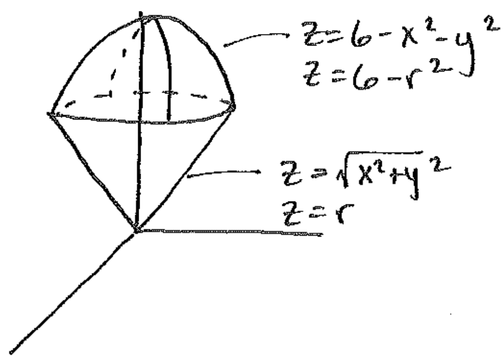
Cylindrical:

$$x^2+(y-1)^2=1$$
$$x^2+y^2-2y+1=1$$
$$r^2-2r\sin\theta=0$$
$$r(r-2\sin\theta)=0$$
$$\cancel{r=0} \quad r=2\sin\theta$$

$$x^2+(y-2)^2=4$$
$$x^2+y^2-4y+4=4$$
$$r^2-4r\sin\theta=0$$
$$r(r-4\sin\theta)=0$$
$$r\neq 0 \quad r=4\sin\theta$$

$$\int_0^{\pi}\int_{2\sin\theta}^{4\sin\theta}\int_0^{r^2+3} r\,dz\,dr\,d\theta$$

⑦



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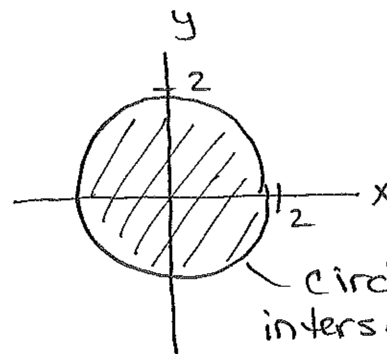
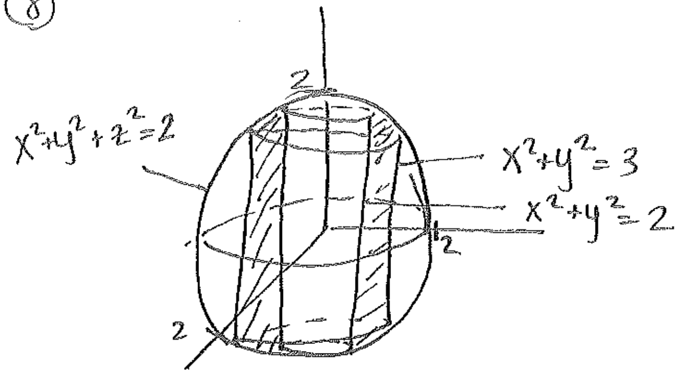


circle is the intersection of the surfaces:

$$6-r^2=\sqrt{r^2}$$
$$6-r^2=r$$
$$r^2+r-6=0$$
$$(r+3)(r-2)=0$$
$$r\neq -3 \quad r=2$$
$$(r\geq 0)$$

Cylindrical: $$\int_0^{2\pi}\int_0^{2}\int_r^{6-r^2} r\,dz\,dr\,d\theta$$

(8)



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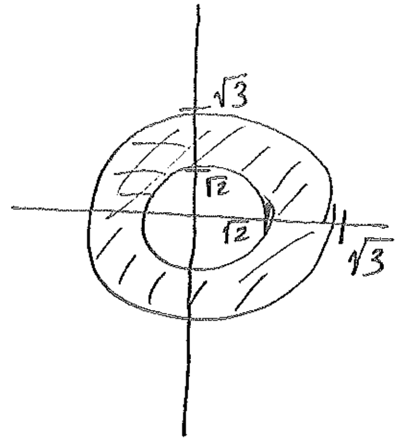


region inside the sphere & between the 2 cylinders

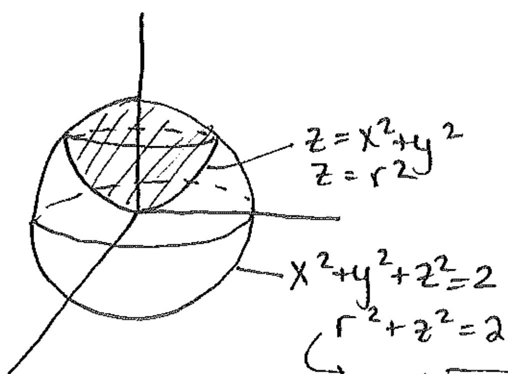
Cylindrical: $x^2+y^2+z^2=2$

$$r^2+z^2=2$$

$$z=\pm\sqrt{2-r^2}$$

$$\int_0^{2\pi}\int_{\sqrt{2}}^{\sqrt{3}}\int_{-\sqrt{2-r^2}}^{\sqrt{2-r^2}} r\,dz\,dr\,d\theta$$

(9)



$$r^2+z^2=2$$

$$z=\pm\sqrt{2-r^2}$$

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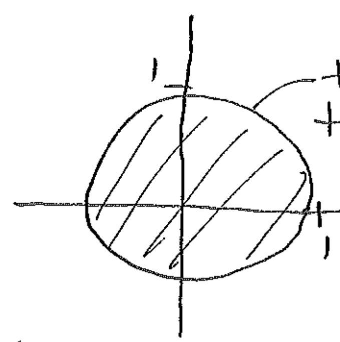

the circle is the intersection of the 2 surfaces

Cylindrical:

$$\int_0^{2\pi}\int_0^{1}\int_{r^2}^{\sqrt{2-r^2}} r\,dz\,dr\,d\theta$$

$$r^2+(r^2)^2=2$$

$$r^2+r^4=2$$

$$r^4+r^2-2=0$$

$$(r^2+2)(r^2-1)=0$$

$$r^2\neq-2 \qquad r^2-1=0$$

$$r^2=1$$

$$r=1$$

$(r\geq 0$, so $r\neq-1)$