Worksheet 19 Solutions

(3) area(R) = 3

$$\frac{1}{\cancel{3}}\int_0^1\int_0^3 \cancel{3}e^y\sqrt{x+e^y}\,dy\,dx \qquad u = x+e^y, \quad du = e^y\,dy$$

$$= \int_0^1 \sqrt{u}\,\Big|_{y=0}^{y=3} dx$$

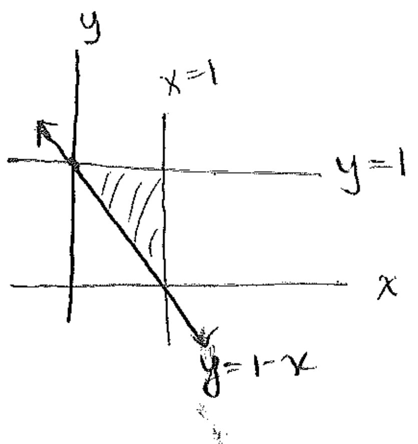
$$= \int_0^1 \sqrt{x+e^y}\,\Big|_0^3 dx = \int_0^1\left(\sqrt{x+e^3}-\sqrt{x+1}\right)dx$$

$$= \frac{2}{3}(x+e^3)^{3/2} - \frac{2}{3}(x+1)^{3/2}\,\Big|_{x=0}^{x=1}$$

$$= \boxed{\frac{2}{3}(1+e^3)^{3/2} - \frac{2}{3}\cdot 2^{3/2} - \frac{2}{3}\cdot e^{9/2} + \frac{2}{3}}$$

(4) $$\int_0^2 (2+6x)\,dx = 2x+3x^2\,\Big|_0^2 = 4+12 = \boxed{16}$$

(5)



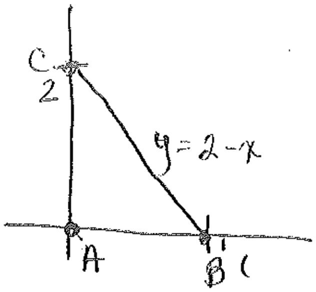
$$\int_0^1\int_{1-x}^1 xy\,dy\,dx = \int_0^1 \frac{y^2}{2}\Big|_{1-x}^1 dx$$

$$= \int_0^1\left(\frac{1}{2} - \frac{(1-x)^2}{2}\right)dx =$$

$$= \frac{1}{2}x + \frac{+(1-x)^3}{6}\Big|_0^1 = \left(\frac{1}{2}+0\right)-\left(0+\frac{1}{6}\right)$$

$$= \frac{1}{2} - \frac{1}{6} = \boxed{\frac{1}{3}}$$

(6)



$$\int_0^1 \int_0^{2-x} (1+3x+y)\, dy\, dx$$

$$= \int_0^1 \left(y + 3xy + \frac{y^2}{2}\right)\Big|_0^{2-x} dx$$

$$= \int_0^1 \left[2 - x + 6x - 3x^2 + \frac{(2-x)^2}{2}\right] dx$$

$$= 2x + \frac{5}{2}x^2 - x^3 - \frac{(2-x)^3}{6}\Big|_0^1$$

$$= \left(2 + \frac{5}{2} - 1 - 0\right) - \left(0 + 0 - 0 - 4/3\right)$$

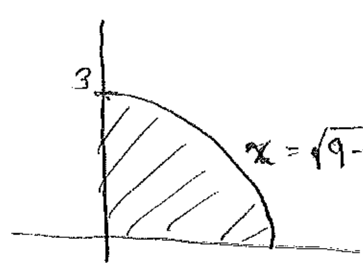
$$= \frac{7}{2} + \frac{4}{3} = \boxed{\frac{29}{6}}$$

(7) $$\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} (x^2+y^2+z^2)\, dz\, dx\, dy$$

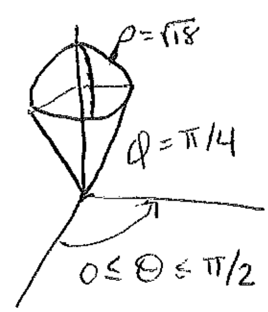
Region bounded by: $z = \sqrt{x^2+y^2}$, a cone: $\phi = \pi/4$

$z = \sqrt{18-x^2-y^2}$, so $x^2+y^2+z^2 = 18$, a sphere: $\rho = \sqrt{18}$

In xy-plane:



so only a quarter of the region between the cone & the sphere:



$$\int_0^{\pi/\cancel{4}2} \int_0^{\pi/4} \int_0^{\sqrt{18}} \rho^2 \cdot \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

$$= \boxed{\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{\sqrt{18}} \rho^4 \sin\phi\, d\rho\, d\phi\, d\theta}$$