MATH 222 (Lectures 1,2,4) **Worksheet 1** **Solutions**

Please inform your TA if you find any errors in the solutions.

1. a Use the identity $\sin^2(\theta) + \cos^2(\theta) = 1$ to show that $\tan^2(\theta) + 1 = \sec^2(\theta)$.

   b For each of the following, circle the correct answer.

$$
\begin{array}{rll}
2\sin(\theta)\cos(\theta) = & \cos(2\theta) & \sin(2\theta) \\
\cos^2(\theta) - \sin^2(\theta) = & \cos(2\theta) & \sin(2\theta) \\
\cos^2(\theta) = & \frac{1}{2}(1+\cos(2\theta)) & \frac{1}{2}(1-\sin(2\theta)) \\
\tan(\arctan(x)) = & 1 & x
\end{array}
$$

Solution:

a Divide on both sides by $\cos^2(\theta)$.

b The correct answers are

$$
\begin{array}{rll}
2\sin(\theta)\cos(\theta) = & \cos(2\theta) & \boxed{\sin(2\theta)} \\
\cos^2(\theta) - \sin^2(\theta) = & \boxed{\cos(2\theta)} & \sin(2\theta) \\
\cos^2(\theta) = & \boxed{\frac{1}{2}(1+\cos(2\theta))} & \frac{1}{2}(1-\sin(2\theta)) \\
\tan(\arctan(x)) = & 1 & \boxed{x}
\end{array}
$$

2. True or False:

   (a) $\frac{d}{dx}(\frac{1}{x}) = \ln x$

   (b) $\frac{d}{dt}\int_0^t \frac{dx}{1+x^2} = \frac{1}{1+t^2}$

   (c) $\sqrt{x^2+9} = x+3$

   (d) The function $f(x) = \frac{1}{x+4}$ is defined for all values of x except for $x = -4$

   (e) $\int e^{(x^3)} dx = e^{(x^3)} + C$

   (f) If $f(x) = x^2 \cdot g(x)$ then $f'(x) = 2x \cdot g'(x)$

Solution:

(a) False.

$$\frac{d}{dx}(\frac{1}{x}) = \frac{d}{dx}x^{-1} = -x^{-2}$$

(b) True.

(c) False.
(d) True.
(e) False. $\frac{d}{dx}e^{(x^3)} + C = 3x^2 e^{(x^3)}$ by the chain rule.
(f) False. By the product rule, we have $f'(x) = 2x \cdot g(x) + x^2 g'(x)$.

3. For each of the following, state whether the object is a function or a number. If it is a function, state what variable it is a function of.

   (a) $\int_1^x e^{\cos(t)} dt$
   (b) $\int_1^3 \sin(s) ds$
   (c) $\int \ln(x) dx$
   (d) $\int_t^{t^3} \ln(\cos(x)) dx$

   Solution:

   (a) This is a function of x.
   (b) This is a number.
   (c) This is a function of x (one of many really).
   (d) This is a function of t.

4. Compute $\frac{d}{dx}\int_x^1 \ln z \, dz$. (Hint: remember the "Fundamental Theorem of Calculus".)

   Solution: The key point is that you don't actually need to compute an antideriviatve of $\ln z$. Namely, just imagine that we have a function $F(z)$ which is an antiderivative of $\ln z$, i.e. a function $F(z)$ where $F'(z) = \ln z$. Then

$$\begin{aligned} \frac{d}{dx}\int_x^1 \ln z \, dz &= \frac{d}{dx}\left[F(z)\right]_{z=x}^{z=1} \\ &= \frac{d}{dx}\left(F(1) - F(x)\right) \\ &= 0 - F'(x) \\ &= -\ln x. \end{aligned}$$

5. Compute $\int e^x \sin(2\pi e^x) dx$

   Solution:

$$\begin{aligned} \int e^x \sin(2\pi e^x) dx &= \int \frac{1}{2\pi} \int \sin(u) du \qquad\qquad u = 2\pi e^x \quad \frac{1}{2\pi} du = e^x dx \\ &= \frac{-1}{2\pi}\cos(u) + C \\ &= \frac{-1}{2\pi}\cos(2\pi e^x) + C \end{aligned}$$

6. Compute $\int_0^x \left(\int_0^t \cos(s)ds\right) dt$.

Solution: We start off by computing $\int_0^t \cos(s)ds = \sin(s)|_0^t = \sin(t) - \sin(0) = \sin(t)$. Then

$$\begin{aligned}\int_0^x \left(\int_0^t \cos(s)ds\right) dt &= \int_0^x \sin(t)dt \\ &= -\cos(t)|_0^x = -\cos(x) - (-\cos(0)) \\ &= 1 - \cos(x)\end{aligned}$$

7. Define $f(x) = \ln(x)\sin(x) + \sqrt{x^4 + x^2}$. Compute $f'(x)$.

Solution:

$$\begin{aligned}f'(x) &= \frac{1}{x}\sin(x) + \ln(x)\cos(x) + \frac{1}{2\sqrt{x^4 + x^2}}\left(4x^3 + 2x\right) \\ &= \frac{1}{x}\sin(x) + \ln(x)\cos(x) + \frac{1}{2|x|\sqrt{x^2 + 1}}\left(4x^3 + 2x\right)\end{aligned}$$