

Name: _____

Solve the following problems.

1. Find the general solution to the differential equation

$$\frac{1}{2x} \frac{dy}{dx} = y + e^{x^2}$$

2. Find a particular solution to the differential equation

$$\begin{aligned} \frac{1+x^3}{3x^2} \frac{dy}{dx} &= 1 - y(x) \\ y(1) &= 2 \end{aligned}$$

3. A tank starts with 100 liters of water and 1,000 bacteria in it. For now we assume the bacteria do not reproduce. Let $B(t)$ be the number of bacteria in the tank as a function of time, where t is in hours. For each of the situations below, write down a first order differential equation satisfied by $B(t)$, of the form $B' = f(t, B)$. You **do not** need to solve it.

(a) A little goblin is pouring bacteria into the tank at a rate of 2015 bacteria per hour.

- (b) Like part (a), but we are also draining the tank at a rate of 3 L/hr.
- (c) Like part (b), but now the bacteria are reproducing. This is a strain of bacteria which, if left alone, will double its population every hour.

4. Find an exact solution to the following initial value problem, then use Euler's method with step size $\Delta x = .1$ to estimate $y(.2)$

$$\frac{dy}{dx} = 2xy + x$$
$$y(0) = 0$$

5. $y(x)$ is a function satisfying $y'' = y' + y + x$, $y(0) = 1$ and $y'(0) = 2$. Approximate $y(0.2)$ using Euler's method with step size 0.1. You will need to modify the version of Euler's method we learned in class to handle a second-order equation like this one. I got $y(0.2) \approx 1.43$.